Dimension counts for singular rational curves

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Rational curves are essential tools for classifying algebraic varieties. Establishing dimension bounds for families of embedded rational curves that admit singularities of a particular type arises arises naturally as part of this classification. Singularities, in turn, are classified by their value semigroups. Unibranch singularities are associated to numerical semigroups, i.e. sub-semigroups of the natural numbers. These fit naturally into a tree, and each is associated with a particular weight, from which a bound on the dimension of the corresponding stratum in the Grassmannian may be derived. Understanding how weights grow as a function of (arithmetic) genus g, i.e. within the tree, is thus fundamental. We establish that for genus g \leq 8, the dimension of unibranch singularities is as one would naively expect. Multibranch singularities are far more complicated; in this case, we give a general classification strategy and again show, using semigroups, that dimension grows as expected relative to g when g \leq 5. This is joint work with Lia Fusaro Abrantes and Renato Vidal Martins.