On the automorphisms of moduli spaces of curves

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The stack $\bar\mathcal{M}_{g,n}$, parametrizing Deligne-Mumford n-pointed genus g stable curves, and its coarse moduli space $\bar M_{g,n}$ have been among the most studied objects in algebraic geometry for several decades. B. Hassett introduced new compactifications $\bar\mathcal{M}_{g,A}$ of the moduli stack $\mathcal{M}_{g,n}$ and $\bar M_{g,A}$ for the coarse moduli space $M_{g,n}$ by assigning rational weights $A = (a_{1},...,a_{n})$, $0< a_{i} <= 1$ to the markings. In particular the classical Deligne-Mumford compactification arises for $a_1 = ... = a_n = 1$. These spaces appear as intermediate steps of the blow-up construction of $\bar M_{0,n}$ developed by M. Kapranov, and in higher genus they may be related to the Log Minimal Model Program on $\bar M_{g,n}$. In this seminar we deal with fibrations and automorphisms of $\bar\mathcal{M}_{g,n}$ and of these Hassett spaces. For instance, we will prove that $Aut(\bar\mathcal{M}_{g,n})\cong Aut(\bar M_{g,n})\cong S_n$ for any g,n such that $2g-2+n\geq 3$. Finally, we will prove that over any field $\bar M_{0,n}$ is rigid, meaning that it does not admit non-trivialinfinitesimal deformations, and we will apply this result to study the automorphism group of $\bar M_{0,n}$ in positive characteristic.