Néron models of Picard groups by Picard groups

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The Néron model provides a universal extension over a discrete valuation ring R of the degree-zero part $\mathrm{Pic}^0 C_K$ of the Picard group of a smooth curve C_K over K=Frac(R). It is natural to exploit the (relative) Picard functor $\mathrm{Pic}^0 C_R$ of a regular (semi)stable reduction C_R to describe its Néron model $N(\mathrm{Pic}^0 C_K)$. The group $\mathrm{Pic}^0 C_R$ is not separated in general, but the Néron model $N(\mathrm{Pic}^0 C_K)$ equals $\mathrm{Pic}^0 C_R$ modulo the closure of the zero section of $\mathrm{Pic}^0 C_K$ (Raynaud, 1970). In some very special cases, we obtain $N(\mathrm{Pic}^0 C_K)$ without passing through the quotient by simply singling out the identity component of $\mathrm{Pic}^0 C_R$, that is the group of line bundles of degree 0 on all irreducible components. In general, the quotient does not possess a similar modular interpretation but this talk shows that, as we adopt a stack-theoretic stable model of C_K, the Néron model does represent a separated Picard functor of degree-0 line bundles on all irreducible components as soon.