CHARACTERIZATIONS AND INTEGRAL FORMULAE FOR GENERALIZED m-QUASI-EINSTEIN METRICS

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Abstract. The aim of this talk, which is a joint work with E. Ribeiro, is to present some structural equations for generalized m-quasi-Einstein metrics. We recall that a complete Riemannian manifold (M^n, g), is a *generalized m-quasi-Einstein metric* if there exist smooth functions f, and λ on M, such that

$$Ric + \nabla^2 f - \frac{1}{m} df \otimes df = \lambda g, \tag{0.1}$$

where Ric denotes the Ricci tensor of $(M^n,\, g)$, while ∇^2 and $\otimes$ stand for the Hessian and the tensorial product, respectively, and $0 < m \leq \infty$ is an integer. We shall show that an Einstein manifold M^n, $n \geq 3$, carrying such a structure is a space form with a well defined potential f. For instance, for the Euclidean sphere $(\mathbb{S}^n, g_0)$, $n \geq 3$, we deduce that $f = -m \ln\big(\tau - \frac{h_v}{n}\big)$, where τ is a real parameter lying in $(1/n, +\infty)$ and h_v is some height function with respect to a fixed unit vector $v \in \mathbb{S}^n \subset \mathbb{R}^{n+1}$. It is important to point out that if $m = \infty$ and λ is constant, equation (0.1) reduces to one associated to a gradient Ricci soliton as well as if λ is only constant, it corresponds to m-quasi-Einstein metrics that are exactly those n-dimensional manifolds which are the base of an $(n+m)$-dimensional Einstein warped product. The special case of 1-quasi-Einstein metrics satisfying $\Delta e^{-f} + \lambda e^{-f} = 0$ are more commonly called *static metrics*, they have connection with scalar curvature, the positive mass theorem and general relativity. In addition, considering $m = \infty$ in equation (0.1) we obtain the almost Ricci soliton equation. Finally, we shall deduce the next formula for the Laplacian of the scalar curvature R of a generalized m-quasi-Einstein metric:

$$\begin{aligned} \frac{1}{2}\Delta R &= (n-1)\Delta\lambda + \lambda\Delta f - |\nabla^2 f - \frac{\Delta f}{n} g|^2 - \frac{n}{2}\langle \nabla f, \nabla\lambda\rangle + \langle \nabla f, \nabla R\rangle \\ &\quad + \Big\{\frac{m-2}{2m}\Big\}\langle \nabla f, \nabla \Delta f\rangle - \Big\{\frac{m+n}{nm}\Big\}(\Delta f)^2 + \frac{1}{m} div\,\big(\nabla_{\nabla f}\nabla f\big), \end{aligned}$$

which enables us to derive some integral formulae for the compact case, deducing some rigidity results.

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