ABSTRACTS

Instantons and Lattice Monte Carlo Methods in Turbulence

David Mesterhazy (University of Bern)

One of the fundamental open problems in the theory of turbulence, is to understand the statistics of small-scale fluctuations from first principles. The competition of vastly different scales makes this problem a hard one to tackle, which affects experimental, numerical, as well theoretical efforts to understand high-Reynolds number flow. In such a situation it is useful to have a model system at hand that shares essential properties with the original problem and offers for a clear physical picture.

An example that has featured prominently over the years is the one-dimensional random-force-driven Burgers equation. Previous studies of Burgers' equation have significantly contributed to our understanding of anomalous scaling and intermittency largely based on ideas from field theory and direct numerical simulations. The latter have allowed for a precise determination of the scaling spectrum for moments of velocity differences in the presence of a power-law forcing that yields a Kolmogorov energy spectrum.

Here, we present a lattice Monte Carlo approach that starts from an effective field-theory action for Burgers' equation driven by a generic Gaussian forcing and highlights those saddle-point configurations associated with the tails of the probability distribution. We show that these configurations give the dominant contributions to the anomalous scaling of high-order moments. Our method yields a clear and quantitative understanding of the relevant statistical structures in the flow that are associated to the strong, intermittent fluctuations characteristic of fully developed turbulence.

Vorticity instanton renormalized by fluctuations

Gregory Falkovich (Weizmann Institute of Science, Israel)

For the direct cascade of steady two-dimensional (2D) Navier-Stokes turbulence, we derive analytically the probability of strong vorticity fluctuations. That derivation is done analytically in two steps: 1) explicitly averaging over fluctuations and obtaining renormalized instanton equations, 2) solving instanton equations. The vorticity PDF thus found has exponential tails and is self-similar, that is, it can be presented as a function of a single, in distinction from other known direct cascades.

Population dynamics with a feedback control

Takahiro NEMOTO (ENS Lyon, France)

When the population dynamics method is applied to low-noise dynamics, a large number of clones is required. This is very important, especially when we want to apply the method to interesting dynamical systems, such as the Shell model for turbulence. In this talk, with a basic Langevin equation, we are explaining the problems related to low-noise dynamics, and show an alternative method that can avoid these problems.

Instantons, zero modes and fluctuations in the Kraichnan model for turbulent advection

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Self-similar instanton solutions of the stochastic dynamics of the full passive scalar field in the Kraichnan model of turbulent advection are arguably good candidates for describing the coherent motion of fluid particles leading to the production of strong gradients of the scalar and intermittency properties. There exists a whole continuum of them, with scaling index ranging between 1 (regular field) and 0 (discontinuous field). We show that they can be set in correspondence (by a Legendre transform) with the so-called zero modes which come out when one considers the evolution of clouds of a finite number of Lagrangian particles and are known, since the late 90s, to constitute the good mathematical framework for understanding and quantifying anomalous scaling in the Kraichnan model.

Sticking to the classical level of approximation in the instanton approach, *i.e.*, concentrating on trajectories realizing an extremum of the action, leads clearly to an overestimation of scaling exponents of the structure functions of the scalar. To get further and incorporate the effect of fluctuations, we rather use the instanton solutions as basic building blocks of trial many-body wave functions for zero modes. This approach gives surprisingly good estimates of scaling exponents even at low statistical orders (at least in 2d and for the values of parameters investigated so far), but still leaves mysterious their ultimate behaviour at large orders where saturation is expected.

Large deviation theory and the Eyring–Kramers formula for non gradient dynamics. Applications to abrupt transitions for turbulent atmosphere jets.

Freddy Bouchet, Julien Reygner, Eric Simonnet, and Tomas Tangarife

Many natural and experimental turbulent flows display a bistable behavior: rare and abrupt dynamical transitions between two very different subregions of the phase space. The most prominent natural examples are probably the Earth magnetic field reversals (over geological timescales), the Kuroshio bistability, or the Dansgaard-Oeschger events that have affected the Earth climate during the last glacial period, and are probably due to several attractors of the turbulent ocean dynamics. Recent results show that similar bistability occur also in the turbulent dynamics of atmosphere jets, for instance on Jupiter troposphere. Those abrupt transitions are extremely rare events that change drastically the nature of the flow and are thus of paramount importance.

We will present the proof of a new formula for the transition rates between two basins of attraction of dynamical systems with weak noises, for non gradient dynamics, in arbitrary dimensions. This formula extend both the large deviation results of Freidlin-Wentzell theory (by computing the prefactor of the large deviation estimate), and the Eyring-Kramers formula valid only for gradient dynamics. We discuss applications to turbulent flows. We will discuss the metastable turbulent dynamics of atmosphere jets, described by the stochastic barotropic quasi-geostrophic model, the simplest model for Jupiter's troposphere. We will discuss analytical results (based on averaging and large deviations) and numerical results (based on the adaptive multilevel splitting algorithm).

Dynamics after blowup: a universal route to stochasticity in turbulence models

Alexei Mailybaev (IMPA, Rio de Janeiro)

The talk will start with an overview of the studies of blowup in simple turbulence models and their relation with intermittency in the developed turbulence, including possible extensions to instantons. The new results will be related to understanding of the universal (chaotic and spontaneously stochastic) behavior of inviscid solutions immediately after the blowup. Starting with the Burgers equation and continuing with the Sabra shell model of turbulence (as well as its continuous 1D representation), I will show how the model can be mapped into a chaotic dynamical system in renormalized coordinates and time, leading to a universal traveling-wave probability measure moving from small to large scales in finite time.

Velocity Gradient Fluctuations in a Lagrangian Model of Turbulence

Luca Moriconi
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Lagrangian turbulence can be usually formulated in terms of a set of non-local and non-linear stochastic differential equations for the evolution of local fluid dynamic observables of interest. In the "Recent Fluid Deformation Closure" (RFDC) model, introduced almost a decade ago by Chevillard and Meneveau [1] for the description of turbulent velocity gradient fluctuations, non-local issues are bypassed by means of a local closure procedure which relies on the deformation kinematics of fluid blobs along lagrangian trajectories. The model's statistical regimes are defined by an effective Reynolds number parameter, and reveal, from the analysis of straightforward numerical solutions, the outset of intermittency as the Reynolds number increases, viz., fat-tailed velocity-gradient probability distribution functions (vgPDFs).

An analytical approach to the RFDC model was carried out in Ref. [2], in the framework of the Martin-Siggia-Rose field theoretical formalism. It turned out that a reasonably good agreement between the analytical and the numerical vgPDFs can be achieved, despite a few simplifying assumptions taken in that investigation, namely, that (i) non-linearity is neglected in the derivation of approximate instanton solutions, and (ii) fluctuations effects are assumed to be encoded uniquely by noise-renormalization contributions in the low-frequency limit of the response functions. In the present work we improve upon the previous study and carefully examine the aforementioned hypotheses (i) and (ii). Relying on the perturbative expansion around the instantons, and going beyond pure noise-renormalization, a set of Feynman diagrams is selected and exacty computed, up to second order in the cumulant expansion. Having in mind the fat-tail profiles of intermittenty distributions, the comparision between the analytical and numerical vgPDFs, as well as the model's geometrical statistical features, are then revisited.

References

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Instantons in a Lagrangian model of turbulence

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Large deviations of the velocity field differences are ubiquitous in high Reynolds number turbulence. Those large excursions of the velocity gradient put their fingerprints in the probability distribution function (pdf), where drastic deviations from gaussian behaviour are apparent. The role of the rare events can be revealed by means of the Martin-Siggia-Rose path-integral formulation. In this work we apply this technique to a model of Lagrangian turbulence called the Recent Fluid De- formation Closure (RFDC). This closure comprises a stochastic model of the velocity gradient tensor based on short time correlations in the Lagrangian frame. Within the path-integral formalism the most probable trajectory that leads to an extreme event is called instanton which corresponds to the path that minimises the action. Numerical calculation of instanton of the RFDC model and its contribution to the pdf is accomplished. Analytical results can be obtained in the computation of the pdf of the diagonal component of the velocity gradient.

Large deviation theory and simulation of heat waves in climate models

Francesco Ragone
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One of the main goals of climate science is to characterize the statistics of extreme, potentially dangerous events (e.g. exceptionally intense precipitations, wind gusts, heat waves) in the present and future climate. The application of techniques and algorithms based on large deviation theory to observational datasets and/or climate numerical simulations could help to determine accurately the probability of these events and to study the dynamical processes responsible for their formation. For example, extreme values of European surface temperatures are connected to the occurrence of heat waves, events that have return times ranging from decades to hundreds of years, and whose statistics is thus difficult to compute. Here we perform computations of large deviation functions of European surface temperatures, applying a population dynamics algorithm to ensemble climate simulations with Plasim, a simple but comprehensive general circulation model of the atmosphere. We compare the performances of this method against direct estimates based on long single runs of the model, and we discuss the feasibility of this class of methods for applications with state of the art climate models, as well as the general appeal of using concepts from large deviation theory to study heat waves and other extreme events in the climate system.

Statistics of superfluid turbulence

Victor S. L'vov
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Turbulence in superfluid helium is unusual and presents a challenge to fluid dynamicists because it consists of two coupled, interpenetrating turbulent fluids: the first is inviscid with quantized vorticity, the second is viscous with continuous vorticity. Due to its dual nature, the energy spectra of the normal- and super-fluid turbulent velocities at sufficiently large length scales are thought to be completely different from those of ordinary turbulence and still barely known.

After a brief historical overview I will present experimental, numerical and theoretical results that pave the way to clarify this problem.

Breaking phenomena in incompressible fluids as a route to the Kolmogorov and Kraichnan spectra

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Abstract

It is shown that continuously distributed frozen-in-fluid vector fields can be compressed (in 2D these are di-vorticity lines, in 3D - vorticity lines). Therefore in both 2D and 3D hydrodynamic developed turbulence there exists the breaking phenomenon. In 2D it leads to the formation of sharp vorticity gradients that provides the Kraichnan spectrum. In 3D the breaking of vortex lines is a reason of the formation of vortex pancake structures and appearance of the Kolmogorov behavior.

For two-dimensional turbulence we study the appearance of sharp vorticity gradients and their influence on the turbulent spectra [1]. We have developed the analog of the vortex line representation (VLR) [2] as a transformation to the curvilinear system of coordinates moving together with the di-vorticity lines (di-vorticity is the vector equal to $\nabla \times \omega$ where ω is the vorticity). Compressibility of this mapping can be considered as the main reason for the formation of the sharp vorticity gradients at high Reynolds numbers. In the case of strong anisotropy the sharp vorticity gradients can generate spectra which fall off as k^{-3} at large k resembling the Kraichnan spectrum for the enstrophy cascade. For weak anisotropy the spectrum due to the sharp gradients coincides with the Saffman spectrum [3]: $E(k) \sim k^{-4}$. We have compared the analytical predictions with direct numerical solutions of the two-dimensional Euler/NS equations for decaying turbulence and for forced turbulence as well. We observed that the di-vorticity is reaching very high values and is distributed locally in space along piecewise straight lines. Thus, indicating strong anisotropy and accordingly we find a spectrum close to the k^{-3}-spectrum [1, 4]. In the numerical experiments [4] for the 8192 x 8192 grid points we observe the spectra with strong angular dependence which can be interpreted as a set of jets with their both weak and strong overlapping. The structure functions of second and third orders show a good correspondence to the Kraichnan direct cascade picture with the constant enstrophy flux. Powers ζ_n for higher structure functions grow weaker the linear dependence relative to n demonstrating the intermittency property.

Recent numerical experiments in the framework of the Euler equations for two colliding Lamb vortex dipoles by Orlandi, et al [5] testify to favor of the collapse appearance when the vorticity becomes infinite in a finite time according to the law $(t_0 - t)^{-1}$, the collapse region vanishes like $(t_0 - t)^{1/2}$, and the velocity component parallel to the vorticity blows up proportionally to $(t_0 - t)^{-1/2}$. During the collapse the region of the maximal vorticity represents the pancake-like structure. All these self-similarities can be obtained from the analysis of the singularity while breaking of vortex lines. In the collapse instant the vorticity Ω gets the singularity of the Kolmogorov type: $\Omega \sim x^{-2/3}$ where x coincides with the coordinate along the breaking direction.

Our numerical experiments [7], however, show exponential growth of the vorticity maximum, $|\omega|_{max} \propto \tau(t)^{-1} = e^{t/T_\omega}$, with formation of the pancake structures that is in agreement with the first numerical observations by Brachet, et al. [6]. The pancake thickness ℓ is shown to decrease exponentially: $\sim e^{-t/T_\ell}$. The ratio T_ω/T_ℓ numerically is very close to 2/3 so that $|\omega|_{max} \propto \ell^{-2/3}$, in a full agreement with the Kolmogorov theory. It is shown also that such a dependence, being a simple consequence of the VLR, is defined by asymptotic behavior of the corresponding Jacobian while breaking of vortex lines when $\tau \to 0$. Simultaneously the theory, based on the VLR, predicts that the pancake size vanishes but more slowly than its thickness ℓ, namely, $\sim \tau^{1/2}$. In spite of exponential growth of the vorticity maximum, the vorticity tail behind its maximum occurs to be frozen (independent of time) with the Kolmogorov exponent $|\omega| \sim x^{-2/3}$ where x coincides with the coordinate along the breaking direction. This is the general statement independent of whether the breaking takes place in a finite or infinite time. The only requirement is $\tau(t)$ should tend to zero.

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