

Area Estimates and Rigidity of Non-compact H -Surfaces in 3-Manifolds

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Abstract

Inspired by results for compact surfaces proved in [?] and [?], we prove a rigidity result for non-compact H -surfaces in 3-manifolds.

Let (M, g) be a complete oriented 3-manifold with sectional curvature $K_M \leq -1$. Let $\Sigma \subset M$ be a complete non-compact oriented H -surface, $0 \leq H < 1$, with finite topology and finite area. Then we prove that Σ has finite total curvature and satisfies the area estimate

$$|\Sigma| \leq \frac{2\pi|\chi(\Sigma)|}{1-H^2}. \quad (1)$$

Moreover, if $H \in (0, 1)$, and the equality in equation (??) is valid, then under additional assumptions on Σ and (M, g) , a neighbourhood of the mean convex side of Σ must be locally isometric to a hyperbolic Fuchsian manifold.

Also if Σ is a minimal surface we prove by an counter-example that one does not have rigidity in (M, g) .

References

- [1] J. Espinar, H. Rosenberg: *Area estimates and rigidity of capillary H -surfaces in three-manifolds with boundary*. arXiv:1511.03096
- [2] L. Mazet, H. Rosenberg: *On minimal spheres of area 4π and rigidity*. Comment. Math. Helv. 89 (2014), no. 4, 921-928.