Recent Advances in Mixed-Integer Nonlinear Optimization

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Many optimal design, operational, and planning problems involve both continuous and integer variables. Often these problems also involve nonlinear relationships that model complex physical interactions, giving rise to mixed-integer nonlinear programming problems (MINLPs). MINLPs combine the combinatorial complexity of the discrete decisions with the numerical challenges of the nonlinear and nonconvex functions, giving rise to tough global optimization problems that are not only NP-hard, but sometimes even undecidable. We will review a range of challenging scientific and engineering MINLP applications, including the modeling of the power grid for network expansion or transmission switching, the optimal reloading of nuclear reactors, the optimal response to oil-spill disasters, and the design of nano-photonic divides.

The last 10 years or so have seen a rise in new algorithmic approaches and software packages for solving MINLPs. We will review some of these methods for solving MINLPs and present a new package for solving mixed-integer nonlinear optimization problems, MINOTAUR. The MINOTAUR toolkit is designed to provide a flexible and efficient C++ framework for solving MINLPs. We will show how MINOTAUR enables us to exploit nonlinear structure in the solution process. We also present a new approach to generate tight and computationally tractable convex relaxations based on exploiting group-partial separability of the nonlinear functions. We demonstrate this approach in the context of two classes of powerful relaxation techniques: semi-definite relaxations and the reformulation-linearization technique. In both cases, we derive tight relaxations that can be solved orders of magnitude faster. We conclude with an outlook of the computational challenges and further research opportunities in MINLP.