

# Functional Ito Calculus, Path-dependence and the Computation of Greeks

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Dupire's functional Ito calculus provides an alternative approach to the classical Malliavin calculus for the computation of sensitivities, also called Greeks, of path-dependent derivatives prices. In this paper, we introduce a measure of path-dependence of functionals within the functional Ito calculus framework. Namely, we consider the Lie bracket of the space and time functional derivatives, which we use to classify functionals according to their degree of path-dependence. We then revisit the problem of efficient numerical computation of Greeks for path-dependent derivatives using integration by parts techniques. Special attention is paid to path-dependent functionals with zero Lie bracket, called weakly path-dependent functionals in our classification. We then derive the weighted-expectation formulas for their Greeks, that was first derived using Malliavin calculus. In the more general case of fully path-dependent functionals, we show that, equipped with the functional Ito calculus, we are able to analyze the effect of the Lie bracket on computation of Greeks. This was not achieved using Malliavin calculus. Numerical examples are also provided.