Pointwise regularity for a parabolic equation with log-term singularity

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Abstract

We study existence and pointwise regularity results for the following parabolic free boundary problem:

$$u_t - \Delta u = \chi_{\{u>0\}} \log u \ \text{ in } \ \Omega \times (0,T],$$

with initial and boundary conditions in some appropriate spaces. The equation is singular along the set $\partial\{u > 0\}$ and the logarithmic nonlinearity does not have scaling properties. Thus, the machinery from regularity theory for free boundary problems, which strongly relies on the homogeneity of the problem, can not be applied directly. We prove that, close to the free boundary, an approximate solution grows at most like $r^2 \log r$. This is the so called super-characteristic growth and its study has intriguing open questions. Our estimates are crucial in the understanding of further analytic and geometric properties of the free boundary.

KEYWORDS: Free boundary, Regularity Theory, Existence, Logarithmic Singularity.

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