Derivative-free optimization: A globally convergent algorithm for unconstrained problems

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Abstract:
Many industrial and engineering applications need to solve optimization problems in which the derivatives of the objective function are unavailable. Nevertheless, they try to avoid unnecessary evaluations in the objective function. The absences of computable derivatives prohibit the use of Taylor models largely used in differentiable problems. Moreover, in general, the optimization without derivatives is not easy, since we attempt to obtain a minimum point with less information. Such situations motivated many researchers to pursuit techniques for derivative-free optimization and to extend differentiable results to algorithms without derivatives. Some interesting work of theory and methods for derivative-free optimization can be found in the work [2] by Conn, Scheinberg, and Vicente.

Powell has a great contribution in this area. The algorithms UOBYQA, BOBYQA and NEWUOA proposed by him in [4, 5, 6] respectively, present good methods for practical purposes. But the non-differentiability of the problems and the practical deals becomes proof of convergence of the algorithms relatively complex. In [3], Powell proposed a simplification version of these works with goal of the theoretical results.

The purpose of this work is to present an explicit trust-region algorithm for minimizing a twice differentiable objective function without using its derivatives. The algorithm is based on the method described by Powell in [3]. In each iteration, a quadratic model of the objective function is constructed by polynomial interpolation with an affine linear set. The algorithm can perform two types of attempts: trust-region and alternative attempts. The trust-region ones are based in [1] and they aim to minimize the model in hope that most of the reduction obtained by the model is inhered by the objective function. The alternative ones aim to improve the geometry of the interpolation set. The objective function is evaluated at just one point on each iteration. So at some iteration the algorithm

performs the three types of attempts until one of these computes the objective function at some point, then this iteration receives the name of this accepted attempt and the index of the iteration is increased. Many properties of this algorithm is obtained. Moreover we prove the global convergence of this algorithm which means that all accumulation point of the generated sequence by the algorithm is stationary.

Key words: trust-region methods, convergence theory, derivative-free optimization, unconstrained minimization.

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