A Gonzaga's Problem

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Abstract

A Gonzaga's problem is the following linear programming problem: from the initial non feasible point $e = [1, 1]^T \in R^2$, solve

$$\begin{array}{lll} (G) & \textit{minimize} & x_1 \\ & \textit{subject to} & x_1 - a x_2 = a \\ & & x_1,\ x_2 \geq 0, \end{array}$$

where $a \geq 10$.

There is an open problem in linear programming (LP), studied and enunciated in Mizuno, Todd and Ye [4], which consists in obtaining an infeasible-interior-point algorithm for the non artificial formulation of the primal-dual LP problem with polynomial complexity $O(\sqrt{n}L)$-iterations, where n is the number of variables of the problem and L is the size of the problem for integer data. With the exception of the $O(\sqrt{n}L)$-iteration homogeneous and self-dual LP algorithm, Ye, Todd and Mizuno [8], which has an artificial formulation for the LP problem, the lower complexity in iterations for the non artificial formulation is $O(nL)$. Recently, several infeasible-interior-point algorithms with full Newton step (see for example, Mansouri and Zangiabadi [1]), based on the work of Roos [6], keep the same complexity. Menezes [2] shows that the difficulty in obtaining $O(\sqrt{n}L)$-iterations for these methods

comes from updating the parameters associated with feasibility and optimality, i. e., $O(nL)$-iteration algorithm decrease these parameters in the same proportion. However, we observed that sometimes we should increase the optimality parameter.

Consider the LP problem (G). We present some graphical outputs for two classes of infeasible-interior-point algorithms for LP in the literature: central surface, based on Stoer [7], Mizuno [3] and Potra [5]; and homogeneous and self-dual, based on Ye, Todd and Mizuno [8].

This work presents a Gonzaga's problem to infeasible-interior-point algorithms for linear programming. The Gonzaga's problem is a simple LP problem with a number of decision variable equal to 2 and a given non interior point, but shows that sometimes is needed that we increase the optimality parameter to obtain $O(\sqrt{n}L)$-iterations algorithms. In this way, it is suggested to have the Gonzaga's problem, in principle, as a test problem for new algorithms.

Keywords: linear programming, infeasible-interior-point algorithm, complexity.

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