Term structure of defaultable bonds, an approach with Jacobi processes

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ABSTRACT

This study provides a novel defaultable term structure model that is capable of capturing negative instantaneous correlation between credit spreads and risk-free rate documented in empirical literature while sustaining the positivity of the default intensity and risk-free rate. Given a multivariate Jacobi (Wright–Fisher) process and a certain functional, we are able to compute the zero-coupon bond prices, both defaultable and default-free, in a relatively tractable way by using the exponential change of measure technique with the help of "carré du champ" operator as well as by using the transition density function obtained from the dual representation of the Jacobi process. The resulting formula involves series involving ratios of gamma functions and fast converging exponential decay functions. The main advantage of the proposed reduced form model is that it provides a more flexible correlation structure between state variables governing the (defaultable) term structure within a relatively tractable framework for bond and derivative pricing. Moreover, in higher dimensions one does not need to rely on numerical schemes related to the differential equations, which may be difficult to handle (e.g multi-dimensional Riccati equations in affine and quadratic term structure frameworks), because the transition density function of the state variables are known. We also illustrate how one can use the proposed model for credit default swap pricing and in a multi-curve setting.

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