Nelson's conjecture revisited

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Resumo/Abstract:
In 1962 Nelson [4] proposed the following conjecture: any divergence free vector field $a : \mathbb{R}^3 \to \mathbb{R}^3$ belonging to $L^p(\mathbb{R}^3)$ for some $p \in [1, \infty]$ generates at most one measure preserving flow. In his seminal paper Aizenman [1] answered negetively this conjecture for $p = \infty$. Note that the vector fields constructed in [1] do not belong to $L^p(\mathbb{R}^3)$ for any $p < \infty$ therefore leaving the question open for $p < \infty$. Recall also that meanwhile uniqueness has been established if a is in some Sobolev (cf. [3]), or, as a refinement, if a is BV (cf.[2]). In this talk I will prove the existence of a bounded divergence free vector field a with compact support (and hence belonging to $L^p(\mathbb{R}^3)$ for any $p \geq 1$) generating two distinct measure preserving flows, thus disproving the conjecture for any $p \geq 1$. As a byproduct I will also establish a non-uniqueness result for the linear transport equation. This is a joint work with Wladimir Neves (UFRJ).

References

[1] Aizenman M., On vector fields as generators of flows: a counterexample to Nelson's conjecture *Ann. Math.* **107** (1978), 287–296.

[2] Ambrosio L., Transport equation and Cauchy problem for BV vector fields, *Invent. Math.* **158** (2004), 227–260.

[3] DiPerna R.J., Lions P.-L., Ordinary differential equations, transport theory and Sobolev spaces, *Invent. Math.* **98** (1989), 511–547

[4] Nelson E., Les écoulements incompressibles d'énergie finie, *Colloques interntionaux du centre national de la recherche scientifique,* **117** (1962), 159.