Semi-linear wave models with power non-linearity and scale-invariant time-dependent mass and dissipation

Wanderley Nunes do Nascimento\textsuperscript{1}, Alessandro Palmieri\textsuperscript{2}, Michael Reissig\textsuperscript{3}

\textsuperscript{1} Institute of Mathematics, Statistic and Scientific Computing IMEEC - UNICAMP, Rua S\‘ergio Buarque de Holanda, 651 Cidade Universitária "Zeferino Vaz" Distr. Barão Geraldo Campinas São Paulo Brasil CEP 13083-859; Announcer supported by FAPESP with the grant 2015/23253-7;

\textsuperscript{2} Faculty for Mathematics and Computer Science Technical University Bergakademie Freiberg Prufstr 9 - 09596 Freiberg - Germany

\textbf{Resumo/Abstract:}

In this talk we will discuss in low space dimensions $n = 1, 2, 3, 4$ the global existence (in time) of small data energy solutions and blow-up behavior of weak solutions to the following semi-linear Cauchy problem with scale-invariant mass and dissipation:

\begin{align*}
  u_{tt} - \Delta u + \frac{\mu_1}{1 + t} u_t + \frac{\mu_2^2}{(1 + t)^2} u = |u|^p \\
  u(0, x) = u_0(x), \quad u_t(0, x) = u_1(x),
\end{align*}

with $(t, x) \in [0, \infty) \times \mathbb{R}^n$, $p > 1$ and $\mu_1 > 0, \mu_2$ are real constants. Our goal is to understand the interplay between $\mu_1$ and $\mu_2$ to prove global existence (in time) of small data energy solutions or blow-up of energy solutions. Scale-invariant mass and dissipation terms are thresholds in the linear theory between non-effective or effective masses and dissipations (see [1], [2], [3]). There is a quite different theory for linear wave models with non-effective or effective mass and dissipation. For this reason, we expect also different results for semi-linear models with power non-linearity. Here different results means that the critical exponent $p_{crit} = p_{crit}(n)$ differs between those for wave models with non-effective or effective mass and dissipation.
Critical exponent means that for small initial data in a suitable space there exists a global (in time) energy solution for some range of admissible $p > p_{\text{crit}}$ and it is possible to find suitable small data such that there exists no global (in time) energy solution if $1 < p \leq p_{\text{crit}}$.

References

