Semi-linear wave models with power non-linearity and scale-invariant time-dependent mass and dissipation

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Resumo/Abstract:
In this talk we will discuss in low space dimensions $n = 1, 2, 3, 4$ the global existence (in time) of small data energy solutions and blow-up behavior of weak solutions to the following semi-linear Cauchy problem with scale-invariant mass and dissipation:

$$\begin{aligned} &u_{tt} - \Delta u + \frac{\mu_1}{1+t} u_t + \frac{\mu_2^2}{(1+t)^2} u = |u|^p \\ &u(0,x) = u_0(x), \quad u_t(0,x) = u_1(x), \end{aligned}$$

with $(t, x) \in [0, \infty) \times R^n$, $p > 1$ and $\mu_1 > 0, \mu_2$ are real constants. Our goal is to understand the interplay between μ_1 and μ_2 to prove global existence (in time) of small data energy solutions or blow-up of energy solutions. Scale-invariant mass and dissipation terms are thresholds in the linear theory between non-effective or effective masses and dissipations (see [1], [2], [3]). There is a quite different theory for linear wave models with non-effective or effective mass and dissipation. For this reason, we expect also different results for semi-linear models with power non-linearity. Here different results means that the critical exponent $p_{crit} = p_{crit}(n)$ differs between those for wave models with non-effective or effective mass and dissipation.

Critical exponent means that for small initial data in a suitable space there exists a global (in time) energy solution for some range of admissible $p > p_{crit}$ and it is possible to find suitable small data such that there exists no global (in time) energy solution if $1 < p \leq p_{crit}$.

References

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