Recent progress in the theory of constant mean curvature surfaces

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Resumo/Abstract:
Joint work with Tinaglia proves that compact disks of constant mean curvature (CMC) 1 embedded in R^3 have curvature estimates away from their boundaries and that there exists a universal bound on the intrinsic radius of such disks. Consequently, any complete, simply-connected embedded surface in R^3 with non zero constant mean curvature must be a round sphere. This work with Tinaglia also implies that complete embedded surfaces in R^3 of positive CMC have bounded second fundamental forms if and only if the have positive injectivity radius. We also prove that a complete embedded surface in R^3 is proper if it has finite topology or positive injectivity radius. In the case of complete hyperbolic three-manifolds X, we prove that a complete embedded surface in X is proper if it has finite topology or positive injectivity and CMC greater than or equal to 1. In contrast to this properness result, joint work with Tinaglia and Coskunuzer shows that in hyperbolic 3-space there exist *nonproper* complete embedded planes of CMC equal to any value $H \in (0, 1)$. We also recently constructed nonproper CMC planes in the Riemannian product $H^2\times$ for any $H \in (0, 1/2)$. My talk ends with an outline of my recent proof with Mira, Perez and Ros of the Hopf Uniqueness Theorem in homogenous 3-manifolds X. This generalization proves that two such spheres of the same CMC are congruent and provides a description of the associated 1-dimensional moduli spaces in terms of the topology and the Cheeger constant of the universal cover of X.