THE INITIAL-BOUNDARY VALUE PROBLEM FOR SOME QUADRATIC NONLINEAR SCHRÖDINGER EQUATIONS ON THE HALF-LINE

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Abstract:
We study the initial-boundary value problem on the right half-line for quadratic nonlinear Schrödinger equation, that is,

$$\begin{cases} i\partial_t u-\partial_x^2 u=N_i(u,\overline{u}), & (x,t)\in(0,+\infty)\times(0,T), i=1,2,3,\\ u(x,0)=u_0(x), & x\in(0,+\infty),\\ u(0,t)=f(t), & t\in(0,T), \end{cases} \tag{1}$$

where $N_1(u,\overline{u})=uu,\ N_2(u,\overline{u})=u\overline{u}$ or $N_3(\overline{u})=\overline{u}\overline{u}$.

The appropriate spaces for the initial and boundary data is motivated by the behavior of solutions to the linear Schrödinger equation in $\mathbb{R}$. Let $e^{-it\partial_x^2}$ linear homogeneous solution group in $\mathbb{R}$ for the Schrödinger equation. The local smoothing inequalities for the operator $e^{-it\partial_x^2}$ is

$$\|\psi(t)e^{-it\partial_x^2}\phi\|_{L_x^\infty H^{\frac{2s+1}{4}}(\mathbb{R})}\leq c\|\phi\|_{H^s(\mathbb{R})}, \tag{2}$$

for example see of [4]. This inequality is sharp in the sense that $\frac{2s+1}{4}$ cannot be replaced by any higher number. We are thus motivated to consider (1) in the setting

$$u_0\in H^s(\mathbb{R}^+) \text{ and } f(t)\in H^{\frac{2s+1}{4}}(\mathbb{R}^+). \tag{3}$$

Our goal is to study (1) under conditions imposed in (3) in order to prove the existence of local solutions with low regularity.

Our main results are summarized in the following Theorem.

Theorem 1. *Let $s_i=-3/4$ for $i=1,3$ and $s_2=-1/4$. For any data $(u_0,f)\in H^s(\mathbb{R}^+)\times H^{\frac{2s+1}{4}}(\mathbb{R}^+)$, with $s\in(s_i,0]$, there exists a positive time $T_i=T_i(\|u_0\|_{H^s(\mathbb{R}^+)},\|f\|_{H^{\frac{2s+1}{4}}(\mathbb{R}^+)})$ and a distributional solution $u_i(x,t)$ of the initial-boundary value problem (1)-(3) with nonlinearity N_i, which is defined in the class $C\big([0,T];\ H^s(\mathbb{R}^+)\big)\cap C\big(\mathbb{R}^+;\ H^{\frac{2s+1}{4}}([0,T]\big)$. Moreover, the map $(u_0,f)\longmapsto u_i$ taking the initial and boundary data to the solution is analytic from $H^s(\mathbb{R}^+)\times H^{\frac{2s+1}{4}}(\mathbb{R}^+)$ to $C([0,T];H^s(\mathbb{R}^+)\cap C(\mathbb{R}^+;H^{\frac{2s+1}{4}}([0,T])$.*

The proof this result involves the method introduced by [1] in their treatment of the generalized Korteweg-de-Vries on the half-line. We also use the ideas contained in [2], [3] and [5]. The main new ingredient is the introduction of a analytic family of boundary forcing operators extending the single

operator introduced by Holmer in [2] and the study of these operators in Bourgain spaces.

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