On The Almost Periodic Homogenization of Non-Linear Scalar Conservations Laws

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Abstract

We deal with the homogenization problem of non-linear scalar conservations laws

$$\begin{cases} \partial_t u_\varepsilon + \nabla_x \cdot (a(\frac{x}{\varepsilon}) f(u_\varepsilon)) = 0, & (x,t) \in \mathbb{R}^{n+1}_+, \\ u_\varepsilon(x,0) = U_0^\varepsilon(x) := U_0(\frac{x}{\varepsilon}, x), & x \in \mathbb{R}^n, \end{cases} \tag{1}$$

where $\mathbb{R}^{n+1}_+ = \mathbb{R}^n \times (0,+\infty)$. The components of the vector field a are assumed to be almost periodic, and the initial data U_0 is also assumed to be almost periodic for a.e. $x \in \mathbb{R}^n$ in its first variable. We present an improvement of a result obtained by Ambrosio and Frid in 2009(Arch. Ration. Mech. Anal. 192, 37-85), that is, assuming only that the flux f is locally Lipschitz, we obtain that the weak limit of the sequence $\{u_\varepsilon\}_{\varepsilon>0}$ of entropy solutions of (1) is the mean value of a function, which we call $U(z,x,t)$, that is the entropy solution of a similar conservation law in the macroscopic variables, but with coefficients depending on the microscopic variables and has the property that the function $z \mapsto \int_{\mathbb{R}^{n+1}_+} U(z,x,t)\varphi(x,t)\,dx\,dt$ is also almost periodic for any $\varphi \in C_c(\mathbb{R}^{n+1}_+)$.