ON THE CAUCHY PROBLEM FOR NONLINEAR INTERACTIONS TYPE SCHRPDINGER

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Abstract:
We study the Cauchy problem associated to the coupled Schrödinger equations, which appears modeling problems in nonlinear optics, namely:

(1)
$$\begin{cases} i\partial_t u(x,t) + p\partial_x^2 u(x,t) - \theta u(x,t) + \bar{u}(x,t)v(x,t) = 0, \quad x \in, \ t \geq 0, \\ i\sigma\partial_t v(x,t) + q\partial_x^2 v(x,t) - \alpha v(x,t) + \frac{1}{2}u^2(x,t) = 0, \quad p,q = \pm 1, \ \sigma > 0, \\ u(x,0) = u_0(x), \quad v(x,0) = v_0(x), \end{cases}$$

where the initial data are considered in the classical Sobolev spaces $(u_0, v_0) \in H^\kappa(\mathbb{R}) \times H^s(\mathbb{R})$.

Well-posedness results for this system, in the periodic case, were obtained by Angulo and Linares in [1]. In this work we develop a local theory for the system, where the regularity (κ, s) of the initial data depends on the different situations of the parameter $\sigma > 0$. Also, we obtain global well-posedness results when $\sigma \neq 2$ and for negative indices $\kappa = s < 0$ included in the local theory developed. Finally, we show some ill-posedness results.

The main references of this work are: [2], [3] and [4].

Remark 1. *The content of this work is part of the author's Ph.D. Thesis at the Universidade Federal de Alagoas under direction of Professor Adán J. Corcho.*

References

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