The Choice Channel of Financial Innovation[*]

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Abstract

Financial innovations in recent decades have vastly expanded investors' portfolio choice. We theoretically analyze the effect of greater choice on investors' savings and asset returns in a model in which investors have possibly heterogeneous beliefs about asset payoffs. Under mild assumptions, we establish a *choice channel* by which greater portfolio choice increases investors' perceived return from saving, and induces them to save more. The effect on asset returns depends on the type of financial innovation. *Portfolio customization* (access to trading risky assets other than the market portfolio) reduces the expected return on every asset. This result is consistent with the decline in the risk-free interest rate since the early 1980s, and is in contrast with the prediction from the "precautionary savings" literature. *Market participation* (improved ability to trade the market portfolio) reduces the risk premium but typically increases the risk-free rate. *Securitization* (relaxation of constraints to issue risk-free debt) dampens the decline in the risk-free rate due to high savings by investors that demand safety. By meeting their choice, securitization induces these investors to save more and exacerbates the global savings imbalances.

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1 Introduction

A key macroeconomic fact of recent decades is the decline in returns of various asset classes. The panels on the left side of Figure 1 illustrate that the short and long term risk-free real interest rates in the US were on an increasing trend before the 1980s but have been declining since the early 1980s. The same trends also apply to the interest rates in most developed economies. The panels on the right side of Figure 1 illustrate the returns on risky assets and display similar trends with some differences. The (model-based) expected return on US stocks has been declining for most of the post-war period, except for an upwards swing in the 1970s. The equity premium—the difference between the expected return on equity and the risk-free rate—also declined in the earlier half of the period, but it appears to be from reasonably constant to slightly increasing in recent decades.[1]

Why are asset returns declining? This question is important not only to understand the evolution of wealth inequality, but also to guide the design of macroeconomic policy. Low interest rates can induce or exacerbate liquidity traps, in which monetary policy is constrained by the zero lower bound (Krugman, 1998; Eggertsson and Woodford, 2003). Recent research has emphasized various factors that might have contributed to low returns. An aging population or rising income inequality in developed economies might have increased the demand for savings, thereby exerting downward pressure on returns (see, for instance Summers, 2014; Eggertsson and Mehrotra, 2014). High demand for (safe) assets by fast-growing emerging markets—known as the savings glut hypothesis—might have also contributed to this pattern (see, for instance, Bernanke, 2005; Caballero, 2006; Caballero, Farhi, and Gourinchas, 2008).[2] In this paper, we supplement these explanations for high savings and low returns with a new rationale: financial innovation that expands investors' portfolio choice. Our analysis can help to explain, among other things, why interest rates have been declining since the 1980s but not in earlier decades.

Our starting point is that financial innovation in the post-war years has vastly increased the trading opportunities in financial markets. The changes have been especially dramatic since the early 1980s. In the mid-1970s, the round trip cost of buying and selling a typical stock was about 5% of the stock price (Turley, 2012), whereas it declined to a few cents in recent years. New financial assets, such as futures, options, and other derivatives, enabled trades that were either impossible or too costly in previous years. While these changes were

[1] These trends in the expected risk premium have also been documented in Blanchard (1993), Jagannathan, McGrattan, and Scherbina (2001), Pástor and Stambaugh (2001), and Fama and French (2002). King and Low (2014) also document the declining trend in real interest rates.

[2] Other rationales include increased uncertainty (see Caballero and Farhi, 2014), a slowdown in productivity, or a reduction in the relative price of investment goods (see Summers, 2014). See Teulings and Baldwin (2014) for a summary of the recent literature on secular stagnation.

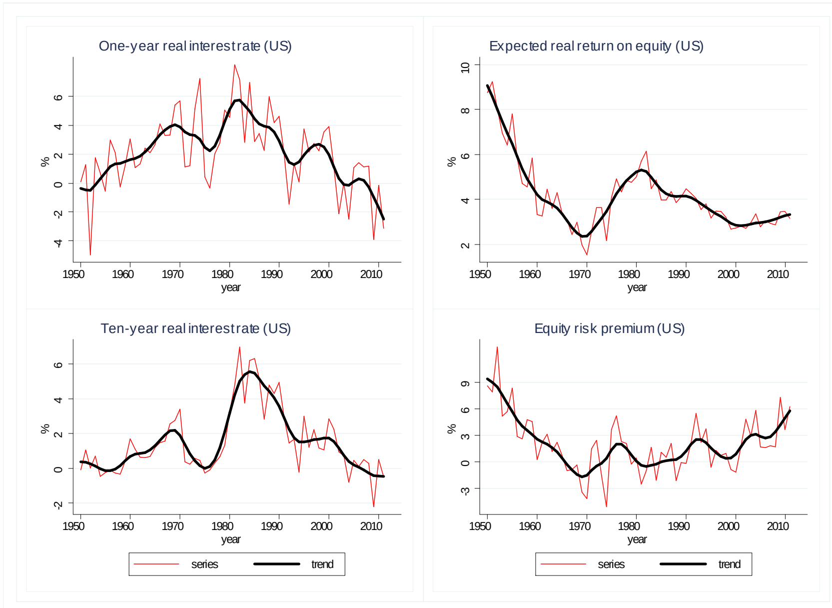

Figure 1: The plots are based on the authors' calculations using the methodology described in Blanchard (1993) and annual returns data from Robert Shiller (available at http://www.econ.yale.edu/ shiller/data.htm). The expected return on equity is calculated by using the dividend yield and the (model- based) expected dividend growth.

driven by multiple factors, the information technology revolution—which accelerated in the early 1980s—arguably played an important role.

These developments have in turn increased households' portfolio choice. In the 1950s, a typical household in the US arguably did not have much choice in constructing her savings portfolio. She could hold bank deposits, and perhaps save in her own house (or durable assets), but she did not have access to many other financial assets. These days, a comparable household can also hold stocks and other risky assets. Figure 4 in Section 6 shows that stock market participation in the US has indeed increased from about 10% of households in the early 1950s to more than 50% (in wealth weighted terms, about 90%) by the end of the 1990s. More importantly, households in recent years can also hold highly customized portfolios. They can choose from a plethora of mutual funds, hedge funds, retirement funds, and ETFs. They can also construct their own portfolios by trading individual stocks, bonds, or a variety of derivatives. Figure 2 shows that mutual funds and exchange traded derivatives, both of which facilitate portfolio customization, have been growing rapidly since the early 1980s (until the recent financial crisis).

Motivated by these observations, we theoretically investigate how financial innovation that increases portfolio choice affects investors' savings and asset returns. In our model,

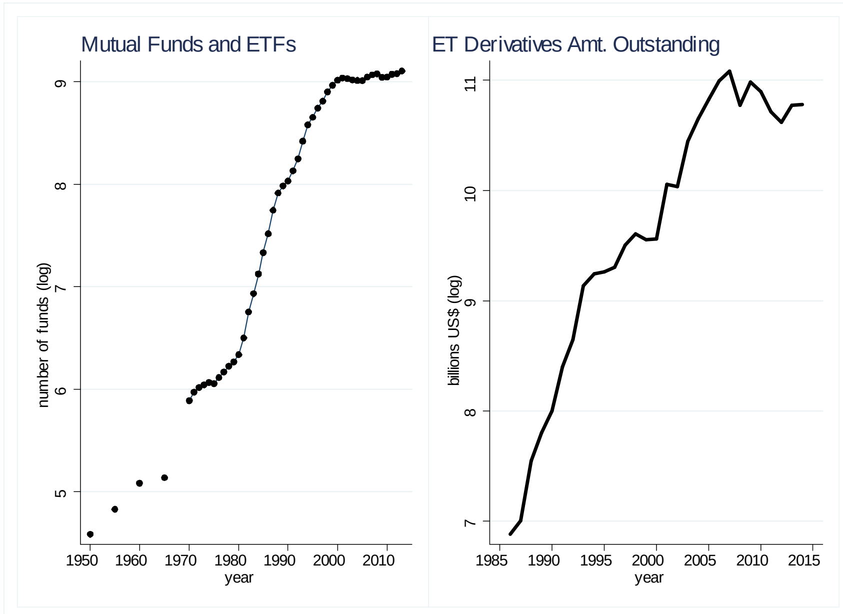

Figure 2: The left panel illustrates the changes in the number of mutual funds and the exchange traded funds in the US (source: Investment Company Institute). The right panel illustrates the changes in outstanding exchange traded derivatives (source: the Bank for International Settlements). Amounts are in constant year 2000 US dollars.

investors with standard Epstein-Zin preferences hold assets to transfer wealth to a future period. Investors optimally choose savings portfolios that consist of the risk-free asset and various risky assets. Each investor has access to the risk-free asset, but investors have limited and (possibly) heterogeneous access to risky assets. We capture financial innovation as an improvement in investors' access sets. We also allow the investors to hold heterogeneous beliefs about risky asset returns, which provides one justification for the portfolio customization illustrated in Figure 2. In benchmark models with homogeneous beliefs, investors would only need one risky asset—namely, the market portfolio—to construct their optimal portfolios. In contrast, investors in our model can demand customized portfolios because they believe those portfolios will yield greater risk-adjusted returns.

Our main result describes the effect of choice on an investor's savings under two assumptions. First, we take the elasticity of intertemporal substitution (EIS) to be greater than one, which we believe is the empirically relevant case in the context of our model (see Section 3). Second, we assume financial innovation does not provide additional benefits in terms of hedging investors' background risks such as income fluctuations—because the investors' initial access set is sufficiently rich to span background risks. In this setting, we show that greater choice induces the investor to save more. This result, which we refer to as *the choice channel* of financial innovation, has a simple intuition. Greater portfolio choice increases the investor's (perceived) certainty-equivalent return on her savings portfolio. This creates substitution and income effects that are similar to those created by an increase in the interest rate. When the EIS is greater than one, substitution effect dominates and the investor increases her savings. With greater choice in financial markets, saving becomes more attractive, and the investor does more of it.

Although the choice channel sounds intuitive, it counters a large "precautionary savings" literature that makes the opposite prediction (see, for instance, Bewley (1977), Huggett (1993), Aiyagari (1994)). This view posits that uninsured background risks induce agents to save for precautionary reasons. The implication is that financial innovation that improves the sharing of background risks should reduce savings, and increase the risk-free interest rate in equilibrium.[3] While we believe background risks are important, especially for economic agents that are net borrowers, incomplete-market models of this type are difficult to reconcile with various pieces of empirical evidence. At the macro level, the interest rate has been declining since the early 1980s in an environment with rapid financial innovation. At the micro level, most investors do not seem to be concerned with background risks when constructing their savings portfolios, as they tend to overinvest in domestic stocks (French and Poterba, 1991), as well as in own company or related stocks (e.g., Benartzi, 2001; Poterba, 2003; Døskeland and Hvide, 2011). They also seem to trade and adjust their portfolios much more frequently than what could be justified by hedging or liquidity needs (Hong and Stein, 2007). These observations can be reconciled with our choice channel when investors' beliefs are sufficiently heterogeneous. Hence, our model identifies a new channel for understanding investors' savings, which can also help to address some of the empirical shortcomings of the precautionary savings literature.

How does the choice channel affect asset returns in general equilibrium? We address this question using a canonical case of our model in which the available financial assets consist of a market portfolio of all cash flows, and several other risky assets in zero net supply. For analytical tractability, we assume a log-normal approximation for portfolio returns (as in Campbell and Viceira, 2002). Greater choice, which increases investors' savings, exerts an upwards pressure on asset prices. However, financial innovation might also generate relative-price effects that interfere with the choice channel. The net effect on prices and returns depends on the type of innovation, which we explore in empirically relevant settings.

Our main general equilibrium result concerns *portfolio customization*, which we capture with improved access to an arbitrary subset of the risky assets other than the market

[3] See Elul (1997) for a formalization and critical evaluation, and Carvajal, Rostek, and Weretka (2012) for a recent application.

portfolio. Under mild symmetry assumptions on investors' beliefs, we show that greater customization reduces the risk-free rate while leaving the risk premia unchanged. In particular, portfolio customization reduces the expected return on each (risk-free or risky) asset. For intuition, imagine financial assets as a forest that contains several trees. The trees could be a methaphor for individual stocks, industries, passive mutual funds with different styles, and active mutual funds with different managers or strategies. Customization enables investors to trade individual trees as opposed to buying or selling claims on the forest. As a response, investors expand their investments in the trees they are optimistic (relative to the average investor), while reducing their positions in other trees. Moreover, for every relatively optimistic investor that buys a particular tree, there are relatively pessimistic investors that sell that tree. Consequently, investors collectively like the forest more, in view of the choice channel, but the relative appeal of individual trees remains unchanged. We show that this logic is general, and implies that greater customization increases the valuation (and reduces the expected return) of each tree in tandem.

We also analyze *market participation*, which we capture with an improvement in investors' access to the market portfolio. This tends to increase asset prices in view of the choice channel, but it also increases the demand for risky assets relative to the safe asset. We find that these relative-demand effects are strong, whereas the choice channel is relatively weak in this context. In particular, while greater participation always reduces the risk premium and the return on the market portfolio, it typically *increases* the risk-free rate for empirically relevant parameters—unlike greater customization.

Our final analysis concerns nontraditional forms of securitization, such as CDOs, which split cash flows from risky securities into safer and riskier tranches. This type of securitization arguably facilitates portfolio choice by meeting the demand for safe assets coming from emerging markets and other sources (see, for instance, Gennaioli, Shleifer, and Vishny, 2012). Figure 4 in Section 6 shows that nontraditional securitization has indeed accelerated in late 1990s, precisely when the emerging market central banks started to accumulate unprecedented levels of safe assets. We capture these developments by expanding our model in a couple of dimensions. We introduce special investors—which we refer to as the EM governments—that have a preference for safe assets in addition to having a relatively high demand for assets. We also introduce securitization as a relaxation of the other investors' constraints to issue safe debt so as to make leveraged investments in risky assets.

Our analysis with EM governments and securitization delivers several insights. First, greater savings by EM governments reduces the risk-free rate, consistent with the conventional wisdom, and reduces the savings of other investors. Hence, the low aggregate savings (or the current account deficits) in the US in recent years can be reconciled with our choice channel once we account for heterogeneity and high savings from other countries. Second, greater savings by the EM governments endogenously increases securitization, which in turn dampens the decline in the risk-free rate. Third, and most relevant for our choice channel, securitization increases the EM governments' savings (relative to the counterfactual). Intuitively, if securitization did not provide additional safe assets, the risk-free rate would decline by more and the EM governments would save less. By meeting their choice, securitization encourages these investors to save more and exacerbates the global savings imbalances.[4]

While our analysis is mainly theoretical, our results are broadly consistent with various trends in asset returns depicted in Figure 1. Between 1950 and the early 1980s, market participation increased considerably but customization was relatively uncommon. This might have contributed to the decrease in the risk premium and the increase in the risk-free rate over this period. Starting in the early 1980s, financial instruments that facilitate portfolio customization have become widespread. This might have contributed to the secular decline of the risk-free rate and other asset returns since the 1980s. Securitization started to accelerate in late 1990s, arguably in response to the increasing demand for safe assets, but collapsed with the recent financial crisis. Its collapse might have contributed to the sharp reduction in the risk-free rate, the increase in the risk premium, as well as the decline in the current account deficits in the US since the financial crisis. These trends of course also have many other contributing factors: our point is that financial innovation and the choice channel work towards explaining the trends (as opposed to exacerbating the empirical puzzles).

The rest of the paper is organized as follows. Section 1.1 discusses the related literature. In Section 2 we present an example that illustrates the choice channel and motivates the rest of our analysis. Section 3 introduces the basic environment and establishes the choice channel along with its comparative statics. Section 4 extends the basic framework into a general equilibrium model with endogenous prices. Section 5 presents our main general equilibrium result on increased customization and returns, along with its various generalizations. Section 6 analyzes the effect of greater participation, and Section 7 analyzes the effect of greater EM savings and securitization. We summarize our findings in a concluding section.

1.1 Related literature

Our paper spans various segments of the literature. As discussed earlier, we contribute to the recent macroeconomics literature on secular stagnation that investigates the sources of low interest rates. We identify financial innovation as a novel factor that can lower the risk-

[4]This is consistent with Justiniano, Primiceri, and Tambalotti (2015), who argue that securitization might have relaxed what they refer to as "lending constraints," and analyze the implications of increased savings for the US housing market.

free rate. A sizeable finance literature also investigates issues related to financial innovation and security design.[5] We focus on the asset pricing implications of financial innovation in an environment with belief disagreements, similar to Fostel and Geanakoplos (2012) and Simsek (2013a).[6] These papers typically take the risk-free rate as given, and characterize how certain types of financial innovations affect the relative price of a single risky asset. In contrast, the equilibrium determination of the risk-free rate has a central role in our analysis.

A recent strand of the financial innovation literature argues that new financial assets can increase portfolio risks in view of belief disagreements and speculation (see, for instance, Simsek (2013b)). Our result on portfolio customization contributes to this literature by analyzing the effect of speculation on investors' savings and asset returns. Our analysis is positive and complements the papers that focus on normative issues surrounding speculation (see, for instance, Brunnermeier, Simsek, and Xiong (2014)). In related work, Kondor and Köszegi (2015) show that financial innovation leads to undersaving in the sense that requiring the investors to save more via more safe assets (while keeping risky asset holdings unchanged) would increase the social welfare. Put differently, while we show that portfolio customization increases saving, Kondor and Köszegi (2015) argue that the adverse welfare effects can be mitigated by encouraging even more saving. There might, however, be other ways to improve welfare in these environments such as taxing financial transactions (Dávila (2015)) or selectively banning speculative trade on new assets (Posner and Weyl (2012)).

Our paper is also related to a large "precautionary savings" literature, which can be divided into two strands according to the sources of risks. The first strand, which we have discussed earlier, focuses on consumption risks driven by background risks such as income fluctuations. A second strand examines the implications of idiosyncratic investment (rate-of-return) risks.[7] This strand emphasizes that enabling firms (or entrepreneurs) to share investment risks can increase aggregate investment. The logic is similar to our choice channel, and relies on a relatively large elasticity of intertemporal substitution. However, our result is different because we analyze households' savings decisions as opposed to firms' investment decisions. Consequently, we delineate conditions under which financial innovation reduces the interest rate, whereas this literature emphasizes that financial innovation can raise investment while still increasing the interest rate (see Angeletos and Calvet (2006) and Angeletos (2007)). In addition, we show that portfolio customization increases savings due to speculation as

[5] A non-exclusive list of additional contributions includes Allen and Gale (1994), Detemple and Selden (1991), Elul (1997), Pesendorfer (1995), Duffie and Rahi (1995), Calvet, Gonzalez-Eiras, and Sodini (2004), and Carvajal, Rostek, and Weretka (2012).

[6] More broadly, our paper is also related to a large literature on asset pricing with heterogeneous beliefs (e.g., Harrison and Kreps, 1978; Scheinkman and Xiong, 2003; Geanakoplos, 2003, 2010; He and Xiong, 2012; Hong and Sraer, 2012).

[7] Key examples include Sandmo (1970), Devereux and Smith (1994), Obstfeld (1994), and Krebs (2003).

opposed to risk sharing.

In parallel and independent work, Guzman and Stiglitz (2015) and Ehling, Gallmeyer, Heyerdahl-Larsen, and Illeditsch (2016) analyze households' consumption and savings decisions in environments with belief disagreements as in our paper. Guzman and Stiglitz (2015) emphasize that disagreements increase investors' perceived wealth, which they refer to as pseudo-wealth, and argue that pseudo-wealth can generate business cycle fluctuations. We emphasize the case in which the substitution effect dominates the wealth effect, and we analyze the longer implications for asset returns as opposed to business cycle fluctuations. Ehling, Gallmeyer, Heyerdahl-Larsen, and Illeditsch (2016) analyze more specifically how belief disagreements about the inflation rate affect the real interest rate, focusing on the case in which the wealth effect dominates (as in Guzman and Stiglitz, 2015).[8]

Our paper is also related to Schmidt and Toda (2016), who analyze investors' consumption and savings decisions in response to bad news. As one example of bad news, they consider the shrinkage of investment opportunities, which leads to a result that is similar to our choice channel. We emphasize financial innovation as opposed to news as the driving force behind the change in investors' choice sets. We also analyze the general equilibrium implications of financial innovation, and obtain several results for asset returns, whereas Schmidt and Toda (2016) focus on the consumption-savings problem.

The part of our paper on participation is related to a large literature that documents limited participation in equity markets and examines its implications for asset prices.[9] Our result that greater participation reduces the risk premium has been noted by this literature, which used low participation as a potential explanation for the historically high levels of the equity premium (see, for instance, Mankiw and Zeldes, 1991; Heaton and Lucas, 1999; Favilukis, 2013). Our result that greater participation increases the risk-free rate (due to a shift of relative demand towards risky assets) appears to be more novel. Basak and Cuoco (1998) demonstrate a version of the result in a dynamic environment in which participants' consumption share evolves endogenously, and nonparticipants are restricted to have log utility. We consider more general preferences than log utility, which makes the choice channel operational and creates a counter-force that tends to lower the interest rate. Our analysis reveals that, for empirically reasonable parameters, the choice channel is relatively weak in this context and the result in Basak and Cuoco (1998) continues to hold.

[8]The idea that belief disagreements increase investors' perceived portfolio returns also appears in Simsek (2013b). The idea that this generates income and substitution effects, and affect investors' savings, appears in Brunnermeier, Simsek, and Xiong (2014). They analyze this in the context of a model by Sims (2009), and use it as an example of how speculation generates behavioral distortions that can be detected (as inefficient) by their welfare criterion.

[9]An incomplete list includes Attanasio, Banks, and Tanner (2002), Brav, Constantinides, and Geczy (2002), Vissing-Jørgensen and Attanasio (2003), Gomes and Michaelides (2008), and Guvenen (2009).

2 A Motivating Example

We first present a simple example that illustrates the choice channel, and provides the motivation for our more general model. Consider an economy with two dates, $t \in \{0, 1\}$, and a single consumption good. At date 1, the economy can be in one of two states, denoted by $\mathbf{z} \in \{shine, rain\}$. There is a single fundamental asset in unit supply, which we refer to as the market portfolio and denote by subscript m. The asset yields payoff only at date 1, denoted by $\overline{\varphi}_m$, which does not depend on the state (for simplicity).

There are two types of investors which we refer to as "optimists" and "pessimists," with heterogeneous prior beliefs about the state $\mathbf{z}$, denoted by $q^{opt}(\mathbf{z})$ and $q^{pes}(\mathbf{z})$. Optimists assign a higher probability to the shine state, $q^{opt}(shine) > q^{pes}(shine)$. Investors are risk-neutral and have a discount factor of one between dates 0 and 1. Thus, they trade financial assets at date 0 to maximize the sum of their expected payoffs at dates 0 and 1. They can take long or short positions in the available financial assets, but they are subject to having nonnegative consumption at each date and state. Investors have large endowments of the asset at date 0, as well as symmetric endowments of the market portfolio, and they have zero endowments of the consumption good at date 1.

First suppose the only available financial asset is the market portfolio. In this case, the equilibrium price of the market portfolio is the same as investors' common valuation,

$$P_m = \overline{\varphi}_m. \tag{1}$$

At this price, investors are indifferent between consuming and saving. The aggregate asset holdings are given by P_m, which clears the asset market.

Next suppose that, thanks to financial innovation, there is a second financial asset in zero net supply that has a positive payoff only in the shine state. The asset is denoted by s and has payoff $\varphi_s(shine) = 1$ and $\varphi_s(rain) = 0$. Together with the market portfolio, this asset completes the financial market and enables investors to take flexible positions on the payoffs in the two states. In equilibrium, optimists hold the payoff in the shine state, as they assign a relatively high probability to this state, $q^{opt}(shine) > q^{pes}(shine)$. Similarly, pessimists hold the payoff in the rain state, since $q^{pes}(rain) > q^{opt}(rain)$. The asset prices are then respectively given by,

$$P_s = q^{opt}(shine) \text{ and } P_m^{comp} = q^{opt}(shine)\,\overline{\varphi}_m + q^{pes}(rain)\,\overline{\varphi}_m. \tag{2}$$

Optimists' and pessimists' asset holdings in equilibrium are given by respectively $q^{opt}(shine)\,\overline{\varphi}_m$ and $q^{pes}(rain)\,\overline{\varphi}_m$.

Comparing Eqs. (1) and (2) shows that financial innovation increases the price of the market portfolio, as well as aggregate asset holdings. Intuitively, providing investors with greater portfolio choice makes saving more attractive, since investors self-select into holding assets or portfolios that they like relatively more. We refer to this effect as *the choice channel* of financial innovation. In equilibrium, greater desire to save translates into higher asset prices.

While this example illustrates the choice channel, it also raises several questions. The example features linear utilities, which implies an infinite elasticity of substitution between date 0 and 1 consumption. One could wonder whether the results are robust to allowing for lower elasticities. The example also features a single asset (before financial innovation), the price of which increases in view of the choice channel. In a more realistic environment with multiple assets, one could wonder how the choice channel affects different asset prices, e.g., whether it increases the price of the risky assets as well as the safe assets. In the rest of the paper, we systematically analyze a more general model, which will enable us to address these questions and deliver additional insights.

3 The Choice Channel and Savings

Consider an economy with two dates, denoted by $t \in \{0, 1\}$, and a single consumption good which will be referred to as a dollar. The uncertainty in this economy is described by the realization of the random variable, $\mathbf{z} \in Z$. There are financial assets denoted by $j \in \{f\} \cup \mathbf{J}$. Each financial asset is a mapping $\varphi_j : Z \rightarrow \mathbb{R}_+$ where $\varphi_j(\mathbf{z})$ denotes the payoff at date 1 if state z is realized. The asset f captures the risk-free asset that makes a constant payment in all states, $\varphi_f(\mathbf{z}) = \overline{\varphi}_f > 0$ for each $\mathbf{z}$. The set $\mathbf{J}$ captures risky assets. We assume (until Section 4) that the state space Z is finite, and the vectors, $\left(\varphi_j(\mathbf{z})\right)_{z \in Z}$ for $j \in \{f\} \cup \mathbf{J}$, are linearly independent so that each asset is non-redundant. Each asset is traded in a competitive market at some price, $P_j > 0$. In this section, we take these prices as given and analyze how financial innovation that expands an investor's choice affects her savings. We endogenize the prices in the next section.

Specifically, consider an investor denoted by the superscript i. The investor starts with some endowment of the consumption good at date 0, denoted by $Y_0^i > 0$, as well as some positions on financial assets, $\left\{x_{-1,j}^i\right\}_j$. We denote the value of financial assets by $W_0^i = \sum_j x_{-1,j}^i P_j$. Thus, her financial wealth at date 0 is $Y_0^i + W_0^i$. The investor also receives some endowment of the consumption good in state $\mathbf{z}$ of date 1, denoted by $L^i(\mathbf{z})$, which can be thought of as her labor (or other non-financial) income. The investor chooses her consumption and total asset holdings at date 0, denoted by C_0 and A_0, as well as positions in financial assets, denoted by $\left\{x_j^i\right\}_{j\in\{f\}\cup J^i}$, to solve,

$$\begin{aligned}
& \max_{C_0, A_0 \geq A_0^{i,\min}, \{x_j\}_{j\in\{f\}\cup J^i}} U_0^i \left(C_0, \left(C_1\left(\mathbf{z}\right)\right)_Z\right) && (3) \\
\text{s.t.} \quad & C_0 + A_0 = Y_0^i + W_0^i \text{ where } A_0 = \sum_{j\in\{f\}\cup J^i} P_j x_j, \\
\text{and} \quad & C_1\left(\mathbf{z}\right) = L^i\left(\mathbf{z}\right) + \sum_{j\in\{f\}\cup J^i} x_j \varphi_j\left(\mathbf{z}\right) \text{ for each } \mathbf{z} \in Z.
\end{aligned}$$

Here, $C_1\left(\mathbf{z}\right)$ denotes the investor's financial wealth in state $\mathbf{z}$ of date 1, which she consumes since there is no subsequent period. The second line captures her budget constraint at date 0 in terms of consumption and asset holdings. We assume $A_0 \geq A^{i,\min}$ (where $A^{i,\min} \in [-\infty, 0]$) so as to allow for an exogenous borrowing constraint as in Aiyagari (1994). The investor allocates her portfolio among various assets in J^i. Note that the investor can take unrestricted long or short positions. The last line illustrates that she receives returns from these assets in the next period.

We assume the investor has recursive Epstein-Zin preferences, given by,

$$\begin{aligned}
U_0^i &= \frac{C_0^{1-1/\varepsilon^i} - 1}{1 - 1/\varepsilon^i} + \beta^i \frac{\left(V_1^i\right)^{1-1/\varepsilon^i} - 1}{1 - 1/\varepsilon^i}, && (4) \\
\text{where } V_1^i &= \left(E^i\left[C_1\left(\mathbf{z}\right)^{1-\gamma^i}\right]\right)^{1/\left(1-\gamma^i\right)}.
\end{aligned}$$

Here, the parameter ε^i captures the investor's elasticity of intertemporal substitution (EIS), which will play a central role for our analysis. The parameter γ^i captures the coefficient of relative risk aversion. The variable, V_1^i, captures the certainty equivalent of future consumption. The special case, $\varepsilon^i = 1/\gamma^i$, corresponds to time separable CRRA preferences. The expectations operator, $E^i[\cdot]$, is taken with respect to the investors' belief about the aggregate state. The superscript i on the investor's expectation operator emphasizes that we allow for heterogeneous beliefs. We also assume the beliefs are dogmatic in the sense that investors do not change their beliefs after they observe the prices (formally, investors know each others' beliefs, and thus, they agree to disagree). We will use belief disagreements of this type to capture investors' demand for customized assets.

We also make the following assumption regarding the investor's background risks.

Assumption 1. For each i, there exists scalars, $\left\{l_j^i\right\}_{j\in\{f\}\cup J^i}$, such that $L^i\left(\mathbf{z}\right) = \sum_{j\in\{f\}\cup J^i} l_j^i \varphi_j\left(\mathbf{z}\right)$ for each $z \in Z$.

The assumption holds when the investor's future endowment is constant. It is also satisfied if the investor's future endowment is perfectly correlated with a combination of the risky assets in her access set.[10] Hence, financial innovation that expands the set J^i does not bring any additional benefits in terms of sharing background risks. Appendix A.1 illustrates that, when this assumption is dropped and several other strong assumptions are added, financial innovation that fully completes the market reduces agents' savings. Intuitively, in that alternative setup, the investor has some precautionary savings motive. Financial innovation enables her to hedge her background risks and mitigates the precautionary savings motive. This precautionary channel of financial innovation is already well understood in the literature. Assumption 1 enables us to abstract away from this channel, and illustrate our novel choice channel.

Under the assumptions we made, there is a unique solution to the investor's problem (3) for a given access set. We next analyze the effect of expanding the investor's access set from some $J^{i,old}$ to a greater set $J^{i,new} \supset J^{i,old}$. Let $\left(C_0^{i,old}, A_0^{i,old}, \left\{x_j^{i,old}\right\}_{j \in \{f\} \cup J^i}\right)$ and $\left(C_0^{i,new}, A_0^{i,new}, \left\{x_j^{i,new}\right\}_{j \in \{f\} \cup J^i}\right)$ denote the solution corresponding to respectively the old and the new access sets. Note that the investor's savings is equal to the change in the value of her assets [cf. Eq. (3)], $S_0^i = A_0^i - W_0^i$. Since asset prices (and thus, W_0^i) are held constant, the savings is determined by the desired asset holdings, A_0^i.

Proposition 1 (Choice Channel)**.** *Suppose Assumption 1 holds and $\varepsilon^i > 1$ (so that the investor's savings is increasing in the interest rate). Then, financial innovation increases the investor's asset holdings (and thus, savings), $A_0^{i,new} \geq A_0^{i,old}$, with strict inequality if $A_0^{i,new} > A^{i,\min}$ and $x_j^{i,new} \neq 0$ for some $j \in J^{i,new} \setminus J^{i,old}$.*

The result says that greater portfolio choice induces the investor to save more. Moreover, the inequality is strict as long as the borrowing constraint does not bind for the investor and she takes a nonzero position on some new asset—so that the assets are not completely redundant from her perspective.

We provide a sketch-proof for this result (completed in the appendix), which is also useful to understand the intuition. Suppose that the investor has zero future endowment, $L^i(\mathbf{z})$. As we show in the appendix, this is without loss of generality, in view of Assumption 1, since an investor with non-zero future labor endowment can be hypothetically thought of as selling her endowment and repurchasing assets.

The investor's problem can be split into two parts. Conditional on asset holdings, A_0^i,

[10] For instance, if an individual's income is perfectly correlated with a broad market index portfolio, and she has access to the index portfolio, then Assumption 1 holds.

the investor solves the portfolio problem

$$V_1^i\left(A_0^i\right) = \max_{\{\tilde{x}_j\}_{\{f\}\cup J^i}} \left(E^i\left[C_1\left(\mathbf{z}\right)^{1-\gamma^i}\right]\right)^{1/\left(1-\gamma^i\right)}, \tag{5}$$
$$\text{s.t.} \sum_{\{f\}\cup J^i} P_j x_j = A_0^i \text{ and } C_1\left(\mathbf{z}\right) = \sum_{\{f\}\cup J^i} x_j \varphi_j\left(\mathbf{z}\right).$$

In turn, given the value function $V_1^i(\cdot)$, she chooses her asset holdings, A_0^i, to maximize the standard intertemporal utility function in (4). The result then follows from three observations. First, the portfolio problem is linearly homogeneous, which implies that the value function is linear in asset holdings,

$$V_1^i\left(A_0\right) = R_{ce}^i A_0. \tag{6}$$

We refer to R_{ce}^i as the investor's certainty-equivalent return. Second, and most importantly, financial innovation increases the certainty-equivalent return, $R_{ce}^{i,new} \geq R_{ce}^{i,old}$, because it expands the choice set of feasible portfolios. Third, in the intertemporal problem, a greater risk-adjusted return implies an increase in asset holdings in view of the assumption $\varepsilon^i > 1$.

Intuitively, with greater portfolio choice, the investor's certainty-equivalent portfolio return can only increase. This creates substitution and income effects. On the one hand, the investor finds savings more attractive, which induces her to save more. On the other hand, the investor also feels richer, which induces her to consume more and save less. The substitution effect dominates, and financial innovation increases savings, whenever the EIS is relatively high.

As this intuition suggests, the result can be further generalized. The particular comparative statics we focus on, the expansion of the access set from $J^{i,old}$ to some $J^{i,new}$, does not play an important role beyond ensuring that the investor has greater choice. Any other financial innovation that expands the investor's choice would induce the investor to save more. In fact, financial innovations that expand the investor's return without affecting her choice would also induce her to save more. For instance, a reduction in trading or intermediation costs works in the same direction as our choice channel.

The result requires two key assumptions: the absence of background risks (Assumption 1) and a relatively high elasticity of intertemporal substitution. As we discuss in Section 1.1, we believe the first assumption is reasonable in our context. Likewise, we also believe a relatively high EIS is appropriate for our context. Using different methodologies, empirical studies find a wide range of estimates for the EIS (see Hall, 1988; Blundell, Browning, and Meghir, 1994; Attanasio and Browning, 1995; Vissing-Jørgensen, 2002; Vissing-Jørgensen and Attanasio, 2003; Gruber, 2013). Most of the studies assume that investors with separable or Epstein-Zin preferences fully observe the changes in the interest rate and make optimal decisions. Even though we also make the same assumptions, some of these features are not central for our analysis. What is important is that investors have an asset holding (or saving) function that is increasing in their perceived portfolio return. We believe this assumption is plausible, and we view the assumption, $\varepsilon^i > 1$, as a simple way of generating an increasing asset holding function in our setting. In our numerical simulations below, we use $\varepsilon^i = 2$, which is the estimate provided by Gruber (2013) based on plausibly exogenous variations in the interest rate that come from tax changes.

For which investors is the choice channel stronger? In the appendix, we establish the comparative statics of the choice channel and obtain various additional testable predictions. We find that the choice channel is stronger for wealthier investors. The main reason is that the investors might face a binding borrowing constraint (which we take as exogenous), in which case their saving does not react to choice.[11] The wealthier investors are less likely to face a binding borrowing constraint, so they are more likely to be subject to the choice channel. In addition, wealthier investors start from a greater base level of assets, and thus, their desired asset levels (and savings) react more to an increase in asset returns. We also find that the choice channel is stronger for more-risk-tolerant investors, because the expansion of choice concerns risky assets.

4 General Equilibrium with Endogenous Returns

We next investigate how the choice channel affects asset prices and returns in general equilibrium. To facilitate analytical tractability, we make several simplifying assumptions. In this section, we describe these assumptions and define the equilibrium. In subsequent sections, we characterize the equilibrium in various cases of interest and establish our main general equilibrium results.

There are several types of investors denoted by $i \in \{1, .., |I|\}$, each of which has population mass $n^i \geq 0$. We normalize the total population mass to 1, so that, $\sum_i n^i = 1$. We make the following analogue of Assumption 1.

Assumption 1$^{\text{G}}$. $L^i = 0, \varepsilon^i > 1$, and $A^{i,\min} = -\infty$ for each investor i.

As before, the investors' future endowments are correlated with assets in their access sets. Without loss of generality, we also normalize the future endowments to zero and simplify the notation (see Section 3). We also assume each investor has a relatively high EIS so that

[11] In fact, if financial innovation that expands choice happens alongside with other innovations that relax the borrowing constraint, then the saving by constrained investors might actually decline.

the choice channel is operational. Finally, we assume the investors do not face borrowing constraints (which does not play an important role beyond simplifying the notation).

To endogenize the asset prices, we impose additional structure on asset supplies and payoffs. Each asset $j \in \mathbf{J}$ is in fixed supply denoted by $\eta_j \geq 0$. More importantly, the uncertainty is now described by a $K \times 1$ vector of continuous random variables, $\mathbf{z} = (z_1, ..., z_K)'$ (in particular, the state space is now given by, $\mathbf{Z} = \mathbb{R}^K$). The log payoff of a risky asset $j \in \mathbf{J}$ can be written as a linear combination of the underlying uncertainty,

$$\log \varphi_j(\mathbf{z}) = \mathbf{F}_j' \mathbf{z}, \tag{7}$$

where $\mathbf{F}_j$ is a $K \times 1$ vector. We assume investors' beliefs for $\mathbf{z}$ are normally distributed, and thus, their beliefs for asset payoffs are log-normally distributed.

Assumption 2. Investor i's prior belief for $\mathbf{z}$ has a Normal distribution, $N(\boldsymbol{\mu}_{\mathbf{z}}^i, \Lambda_{\mathbf{z}})$, where $\boldsymbol{\mu}_{\mathbf{z}}^i \in \mathbb{R}^K$ is the mean vector and $\Lambda_{\mathbf{z}}$ is the $K \times K$ positive definite covariance matrix. In addition, the $K \times |\mathbf{J}|$ matrix of asset loadings, $\mathbf{F} = [\mathbf{F}_j]_{j \in \mathbf{J}}$, has full rank.

Note that investors can disagree on the mean of the asset payoffs but they agree on the variance of log payoffs (for simplicity). The full rank assumption ensures that risky assets are not redundant. As we noted before, the belief disagreements are dogmatic. Formally, investors know each others' beliefs and they agree to disagree.

In this section, we find it convenient to work with gross and log asset returns, defined respectively by $R_j(\mathbf{z}) = \varphi_j(\mathbf{z}) / P_j$ and $r_j(\mathbf{z}) = \log R_j(\mathbf{z})$, for each $j \in \{f\} \cup \mathbf{J}$. Assumption 2 implies that the asset returns are jointly log-normally distributed with mean and joint variance respectively given by,

$$E^i[r_j] = \mu_j^i - \log P_j \text{ for each } j, \text{ and } var\left(\{r_j\}_{j \in \mathbf{J}}\right) = \Lambda. \tag{8}$$

Here, $\mu_j^i = (\mathbf{F}_j)' \boldsymbol{\mu}_{\mathbf{z}}^i$ (and $\mu_f^i = \log \overline{\varphi}_f$) denote investor i's belief for the log asset payoffs, and $\Lambda = \mathbf{F}' \Lambda_{\mathbf{z}} \mathbf{F}$ denotes the common variance matrix that is common to all investors. We define the investor's (log) risk premium on an asset perceived by an investor i as,

$$\pi_j^i = \log E^i[R_j] - \log R_f = E^i[r_j] + \frac{\Lambda_j}{2} - r_f \text{ for each } j \in \mathbf{J}. \tag{9}$$

Intuitively, the investor's risk premium depends on the difference between the expected return on the asset and the risk-free rate. Note that different investors can perceive different risk premia in view of the differences in their beliefs for expected returns [cf. (8)]. We define the risk premium on an asset as the average of the perceived premia, $\pi_j = \sum_i \pi_j^i n^i$. By linearity, this is the same as the risk premium perceived by the investor with the average belief about the asset return, $\sum_i n^i E^i[r_j]$ [cf. (9)].

We also define the investor's portfolio return and the log portfolio return, defined respectively by $R_p^i(\mathbf{z}) = C_1^i(\mathbf{z})/A_0^i$ and $r_p^i(\mathbf{z}) = \log R_p^i(\mathbf{z})$. Using the budget constraint in (3), the portfolio return can be written as a linear combination of her asset returns,

$$R_p^i(\mathbf{z}) = \sum_{j\in\{f\}\cup J^i} \omega_j^i R_j(\mathbf{z}), \text{ where } \omega_j^i \equiv x_j^i P_j / A_0^i.$$

Here, ω_j^i denotes the investor's portfolio weight invested in asset j, which sum to one. Thus, the investor can be thought of as choosing her portfolio weights in risky assets, $\boldsymbol{\omega}_{J^i} = (\omega_j)_{j\in J^i}$, with the residual weight, $\omega_f = 1-\sum_{j\in J^i}\omega_j$, invested in the safe asset. Even though each risky asset has a log-normal distribution, the portfolio return can in general have a complicated distribution. For analytical tractability, we assume the investor optimizes her portfolio after applying a log-normal approximation to portfolio returns as in Campbell and Viceira (2002). This approximation becomes exact in the continuous time limit in which the time horizon between dates 0 and 1 disappears. Over shorter horizons, such as a year, this approach results in reasonable portfolio allocations that are close to optimal.

The investor's portfolio problem (5) can then be approximately written as [see Campbell and Viceira (2002) for details],

$$\begin{aligned} r_{ce}^i - r_f &= \max_{\boldsymbol{\omega}_{J^i}=(\omega_j)_{j\in J^i}} \pi_p^i - \frac{\gamma^i \Lambda_p}{2} \\ \text{such that,} \pi_p^i &= \boldsymbol{\omega}'_{J^i}\boldsymbol{\pi}_{J^i}^i \text{ and } \Lambda_p = \boldsymbol{\omega}'_{J^i}\Lambda_{J^i}\boldsymbol{\omega}_{J^i}. \end{aligned} \tag{10}$$

Here, the variable, $r_{ce}^i = \log R_{ce}^i$, denotes the log of investor's certainty equivalent return from asset holdings [cf. Eq. (6)]. The variables, π_p^i and Λ_p respectively denote the investor's (log) risk premium and the variance on her portfolio. The objective function says that the investor trades off the risk premium and variance according to her risk aversion. The second line describes the portfolio risk premium and variance as a function of the risk premia on the assets the investor has access to, $\pi_{J^i}^i = \left(\pi_j^i\right)_{j\in J^i}$, and their variance, Λ_{J^i}.[12]

The portfolio problem has a closed-form solution given by,

$$\boldsymbol{\omega}_{J^i}^i = \frac{1}{\gamma^i}\Lambda_{J^i}^{-1}\boldsymbol{\pi}_{J^i}^i \text{ and } r_{ce}^i = r_f + \frac{1}{2\gamma^i}\left(\boldsymbol{\pi}_{J^i}^i\right)'\Lambda_{J^i}^{-1}\boldsymbol{\pi}_{J^i}^i. \tag{11}$$

[12] To obtain these expressions, imagine that log asset returns followed a joint diffusion process over continuous time with instantaneous drifts $\left\{\mu_j^i\right\}_J$ and volatility Λ (starting at $r_j = 0$ for each j). Then, the log portfolio return would also follow a diffusion process. Its mean and variance can be characterized using Ito's Lemma, which leads to the expressions in the portfolio problem.

Loosely speaking, increasing the risk premia of an asset, while keeping all else constant, tends to shift the investor's portfolio weight towards this asset. Greater risk premium also enables the investor to obtain a greater certainty-equivalent return, which increases her savings. Likewise, keeping risk premia constant, increasing the risk-free rate raises the certainty-equivalent return and savings.

As before, the investor chooses her consumption and savings to maximize the intertemporal utility function in (4) given the certainty equivalent return from her portfolio problem. The solution can be written as

$$A_0^i = a^i\left(r_{ce}^i\right)\left(Y_0^i + W_0^i\right) \text{ where } a^i\left(r_{ce}^i\right) = \frac{\left(\beta^i\right)^{\varepsilon} \exp\left(r_{ce}^i\left(\varepsilon^i - 1\right)\right)}{1 + \left(\beta^i\right)^{\varepsilon} \exp\left(r_{ce}^i\left(\varepsilon^i - 1\right)\right)}. \tag{12}$$

Here, $a^i\left(r_{ce}^i\right)$ describes the investors' effective asset holding as a fraction of wealth. Importantly, it is an increasing function in view of the assumption $\varepsilon^i > 1$, which captures the choice channel (Proposition 1) in the general equilibrium context.

The asset market clearing conditions can then be written as,

$$\eta_j P_j = \sum_{\{i \;\mid\; j \in \{f\} \cup J^i\}} n^i \omega_j^i a^i\left(r_{ce}^i\right)\left(Y_0^i + W_0^i\right) \text{ for each } j \in \{f\} \cup \mathbf{J}, \tag{13}$$

where $W_0^i = \sum_j P_j x_{-1,j}^i$ for each i. The condition says that the supply of each asset j equals its demand, which is determined by investors' savings as well as their asset allocations. To avoid trivial cases, we assume that each asset that is in positive supply, $\eta_j > 0$, lies in at least one investor's access set. We also assume the economy is closed, so that the assets are initially held by the investors in the economy, that is, $\sum_i n^i x_{-1,j}^i = \eta_j$ for each j.

Definition 1 (Equilibrium)**.** *Under Assumptions 1^G and 2, an (approximate) equilibrium,* $\left\{\left(\omega_{J^i}^i, A_0^i\right)_i, P_j\right\}$*, is a collection such that asset returns and premiums are given by* $(8-9)$*, the investors' portfolio weights and certainty-equivalent returns are given by* (11)*, asset holdings are given by* (12)*, and the markets clear [cf. Eq.* (13)*].*

Proposition 8 in the appendix establishes the existence of an equilibrium. At this level of generality, we cannot characterize the equilibrium much further. We thus analyze a canonical case that can accommodate the key aspects of various recent financial innovations.

Assumption 3. There exist K risky assets in total, $\mathbf{J} = \{m, 1, ..., K-1\}$. The asset m is in positive supply, $\eta_m > 0$, while the remaining risky assets, as well as the risk-free asset are in zero net supply, $\eta_j = 0$ for $j \neq m$.

The first part ensures that the risky assets collectively complete the market, since the underlying uncertainty is K dimensional (see Assumption 2). The second part says that there exists a single asset, denoted by m, that represents all of the cash flows in positive supply. This asset, which is typically referred to as *the market portfolio*, enables investors to obtain exposure to all assets in proportion to their market valuations. Its practical counterpart could be broad equity or bond indices that proxy for this type of exposure. The remaining risky assets, $j \in \{1, .., K-1\}$, enable investors to customize their risk exposures. Throughout, we also maintain the following symmetry assumption.

Assumption 4. Investors start with the same endowment of the consumption good, $Y_0^i = Y_0 > 0$ for each i, as well as the risky assets, $x_{-1,j}^i = x_{-1,j}$ for each j and i.

This assumption ensures that investors have the same wealth, $Y_0 + W_0$. Hence, it enables us to abstract away from the effects of financial innovation on wealth redistribution, which is not our focus. In view of this assumption, the wealth shares of different investor groups are captured by the exogenous relative mass parameters, $\{n^i\}_i$.

5 Portfolio Customization

Our main general equilibrium result concerns customization, which we capture with improved access to assets $j \in \{1, .., K-1\}$. The practical counterpart of these assets can be thought of as direct trading of individual stocks and bonds; investment funds that specialize in certain industries or styles; or derivatives such as futures, options, and ETFs. These financial instruments enable investors to construct customized portfolios according to their needs or beliefs. We first analyze the effect of these assets in a benchmark setting that enables us to obtain a sharp result. We then discuss the extent to which the result generalizes beyond the benchmark.

5.1 Customization in a Benchmark Setting

Suppose investors can differ in their market access as well as their beliefs. Formally, investors' types have two dimensions, $\{i = (i_A, \mathbf{i}_B)\}_i$. The sub-type $i_A \in I_A$ captures the variation in investors' market access, while the sub-type $\mathbf{i}_B \in I_B$ (which itself is a vector) captures the variation in beliefs. Investors' beliefs are drawn independently of their market access.[13] For simplicity, investors are identical in all dimensions other than possibly their market access and beliefs.

[13] More specifically, the mass of type $i = (i_A, \mathbf{i}_B)$ investors is given by $n^i = n^{i_A} \times n^{\mathbf{i}_B}$, where n^{i_A} denotes the mass of investors with market access type i_A, with $\sum_{i_A} n^{i_A} = 1$, and $n^{\mathbf{i}_B}$ denotes the mass of investors with belief type $\mathbf{i}_B$, with $\sum_{\mathbf{i}_B} n^{\mathbf{i}_B} = 1$.

The access types are given by, $I_A = \{0, ..., K-1\}$, such that $J^{i_A} = \{m, 1, .., i_A\}$ for each $i_A \in I_A$. Hence, i_A denotes the number of the non-market assets the investor has gained access to (in increasing order). For simplicity, all investors have access to the market portfolio. We discuss the implications of limited market participation in the next section. The focus of this section is increased customization, which we will capture with a shift of mass from a type with less access to one with more access, that is, $\tilde{n}^{i_A^1} = n^{i_A^1} + \Delta n$ and $\tilde{n}^{i_A^0} = n^{i_A^0} - \Delta n$ where $i_A^1 > i_A^0$ and $\Delta n > 0$. In view of the symmetry assumptions, this is equivalent to expanding the access set of some investors to include the new financial assets $j \in \{i_A^0 + 1, .., i_A^1\}$, as in Section 3. The difference is that financial innovation applies to a positive mass of investors, with potential general equilibrium effects.

The belief types are a collection of K-dimensional vectors, $I_B \subset \mathbb{R}^K$. Investors with type $\mathbf{i}_B \in I_B$ have the belief, $\boldsymbol{\mu}_{\mathbf{z}}^{\mathbf{i}_B} = \boldsymbol{\mu}_{\mathbf{z}} + \mathbf{i}_B$, for the underlying uncertainty. Hence, the type describes the deviation of the investor's belief from the average belief denoted by $\boldsymbol{\mu}_{\mathbf{z}}$. We assume that for each type, $\mathbf{i}_B \in I_B$, the opposite type, $-\mathbf{i}_B \in I_B$, also exists and has equal mass,

$$n^{\mathbf{i}_B} = n^{-\mathbf{i}_B} \text{ for each } \mathbf{i}_B \in I_B. \tag{14}$$

This is a mild symmetry assumption on the cross-sectional belief distribution. We also assume the investors do not disagree about the return of the market portfolio,

$$\mathbf{F}'_{\mathbf{m}} \mathbf{i}_B = 0 \text{ for each } \mathbf{i}_B \in I_B, \text{ which implies } \mu_m^{\mathbf{i}_B} = \mu_m. \tag{15}$$

We discuss the role of this assumption later in the section. The upshot of these assumptions is a closed-form characterization of equilibrium, which we present next.

Lemma 1. *Consider the above setting with limited customization of portfolios, full participation in the market portfolio, and belief disagreements that satisfy* (14) *and* (15)*. There exists an equilibrium in which:*

(i) The risk premium on each risky asset satisfies, $\pi_j = \frac{\Lambda_{jm}}{\Lambda_m} \pi_m$*, where* $\pi_m = \gamma \Lambda_m$*.*

(ii) The risk-free rate, r_f*, is the unique solution to*

$$\frac{\eta_m P_m}{Y_0 + \eta_m P_m} = \sum_{i \in I} n^{i_A} n^{\mathbf{i}_B} a\left(r_{ce}^{(i_A, \mathbf{i}_B)}\right), \tag{16}$$

where the certainty equivalent return for an investor with type $(i_A, \mathbf{i}_B)$ *is,*

$$r_{ce}^{(i_A, \mathbf{i}_B)} = r_f + \frac{1}{2\gamma} \frac{\pi_m^2}{\Lambda_m} + \frac{1}{2\gamma} \left(\mathbf{F}'_{J^{i_A}} \mathbf{i}_B\right)' \Lambda_{J^{i_A}}^{-1} \left(\mathbf{F}'_{J^{i_A}} \mathbf{i}_B\right). \tag{17}$$

The first part says that the risk premium on an asset is determined by its "beta" with respect to the market portfolio. It also characterizes the risk-premium on the market portfolio. These are standard asset-pricing conditions that would also obtain in a version of our model without any heterogeneity in beliefs and complete customization. Hence, for the purposes of characterizing the risk premia, or relative asset prices, the unorthodox features of the model—belief heterogeneity and limited customization—can be ignored. In view of the symmetry assumption (14), for every "optimist" whose portfolio shares deviate from the average portfolio share in a particular direction, there is a "pessimist" whose portfolio deviates in exactly the opposite direction. Since belief heterogeneity does not influence investors' portfolio shares on average, it also does not influence relative asset prices. Limited customization does not influence relative prices either because, absent belief heterogeneity, the market portfolio m is sufficient to construct efficient portfolios in equilibrium.

The second part shows that, although belief heterogeneity and limited customization do not affect relative prices, they can influence absolute asset prices. In particular, Eq. (16) is a market clearing condition for all assets. The left hand side captures the valuation of the market portfolio whereas the right hand side captures the total savings, which depend on investors' certainty-equivalent returns. Since the premium on the market portfolio is determined by the first part, this condition can be thought of as determining the risk-free rate. Eq. (17) shows that investors' certainty-equivalent returns depend on their beliefs as well as their degree of their customization, with implications for the risk-free rate. The next result characterizes the pricing implications of greater customization.

Proposition 2 (Customization)**.** *Consider the equilibrium characterized in Lemma 1. Consider financial innovation that increases the scope of customization for some market participants,* $\tilde{n}^{i_A^1} = n^{i_A^1} + \Delta n$ *and* $\tilde{n}^{i_A^0} = n^{i_A^0} - \Delta n$ *where* $i_A^1 > i_A^0$ *and* $\Delta n > 0$. *This change reduces the risk free rate* r_f*, leaves unchanged the average risk premia,* $\{\pi_j\}_{j \in \mathbf{J}}$ *(and relative prices,* $\{P_j/P_f\}$*), and decreases the average expected return on risky assets,* $\{r_f + \pi_j\}_{j \in \mathbf{J}}$ *(increases* $\{P_j\}$*).*

The intuition follows from Eq. (17), which implies that the investor's certainty-equivalent return, $r_{ce}^{(i_A, \mathbf{i}_B)}$, is increasing in the number of available assets, i_A. Hence, consistent with the choice channel [cf. Proposition 1], greater customization increases investors' savings, $a\left(r_{ce}^{(i_A, \mathbf{i}_B)}\right)$. This in turn increases asset prices, generalizing the result in Example 2. The difference is that the current model features many assets, and the price of all assets increase in tandem. The following example numerically illustrates the result using a special case of the model.

Special Case and Numerical Illustration Suppose that the market portfolio satisfies, $\log \varphi_m = z_K$, and the sources of risk, $\{z_1, .., z_K\}$, are linearly independent. Hence, the source, z_K, captures a systematic risk factor, and the remaining sources, $\{z_1, .., z_{K-1}\}$, capture nonsystematic (e.g., idiosyncratic) risk factors that are orthogonal to the market portfolio as well as one another. For simplicity, suppose each investor is optimistic or pessimistic about one nonsystematic risk factor. Specifically, for each $k < K$, there are two belief types, $\mathbf{i}_{B,k}$ and $-\mathbf{i}_{B,k}$ that respectively think that the mean of z_k is given by $\mu_k + \Delta_k$ and $\mu_k - \Delta_k$, whereas they agree on the objective mean of the remaining risk factors. Thus, type $\mathbf{i}_{B,k}$ investors are optimistic about z_k (and only z_k), whereas type $-\mathbf{i}_{B,k}$ investors are pessimistic, and the degree of disagreement is captured by the parameter, $\Delta_k \geq 0$. All investors agree on the objective mean of the systematic risk factor, z_K. Note that investors' beliefs satisfy conditions (14) and (15).

Suppose also that there is one (non-market) asset per nonsystematic risk factor, that is, $\log \varphi_j = z_j$ for $j \in \{1, .., K-1\}$. These assets can be thought of as the nonsystematic component of stocks, bonds, or other similar financial assets. We also assume investors have symmetric access to these assets, that is, $n^{\bar{i}_A} = 1$ for some $\bar{i}_A \leq K-1$ (and $n^{i_A} = 0$ for $i_A \neq \bar{i}_A$). Hence, the parameter, $\bar{i}_A$, captures the total number of non-market assets available to any investor. Using Eq. (17), the investors' certainty-equivalent return can be written as,

$$r_{ce}^{(\bar{i}_A, \mathbf{i}_{B,k})} = r_{ce}^{(\bar{i}_A, -\mathbf{i}_{B,k})} = \begin{cases} r_{ce}^{nonspec} \equiv r_f + \frac{\pi_m^2}{2\gamma\Lambda_m} & \text{if } \bar{i}_A < k, \\ r_{ce}^{spec} \equiv r_f + \frac{\pi_m^2}{2\gamma\Lambda_m} + \frac{\Delta_k^2}{2\gamma\Lambda_k} & \text{if } \bar{i}_A \geq k. \end{cases} \tag{18}$$

Here, $r_{ce}^{nonspec}$ is the "non-speculative" return the investor can obtain by only trading the risk-free asset and the market portfolio, whereas $r_{ce}^{spec} > r_{ce}^{nonspec}$ is the "speculative" return she can obtain by combining the market portfolio with a position in the risk factor, z_k. The investor is able to obtain the greater speculative return only if the asset k is available for trade. Hence, greater customization (greater $\bar{i}_A$) increases investors' certainty equivalent return by allowing more of them to construct customized portfolios with speculative positions. Note also that the investor's return increases in proportion to the square of her perceived Sharpe ratio on the nonsystematic risk factor, $\left(\Delta_k/\sqrt{\Lambda_k}\right)^2$—which helps to assess the quantitative strength of the choice channel.

For a numerical illustration, we use the preference parameters, $\gamma = 5$ and $\varepsilon = 2$, along with a yearly calibration. To mitigate the equity premium puzzle, we consider a relatively high level for the volatility of the market portfolio, $\sqrt{\Lambda_m} = 8\%$.[14] This leads to the risk

[14] The volatility of the consumption growth in the US is around 1%, which leads to the equity premium puzzle (with relatively standard parameters such as $\gamma = 5$). Our calibration with higher volatility can be thought of as capturing factors omitted from our model, e.g., long-run risk, that could help to explain the

premium $\pi_m = \gamma\Lambda_m = 3.2\%$ and the Sharpe ratio $\pi_m/\sqrt{\Lambda_m} = 0.4$. Assuming that the world equity index and the market portfolio are perfectly correlated (otherwise, the equity premium puzzle becomes even deeper), the implied Sharpe ratio is roughly consistent with the Sharpe ratio on the world equity index in dollars in recent decades (see, for instance, Calvet, Campbell, and Sodini, 2007). We also calibrate the growth rate of log output and the discount factor β so that the risk-free rate without any disagreement is equal to its historical average, $r_f = 1\%$ when $\Delta_k = 0$.

The key variable for our analysis is the perceived Sharpe ratio on nonsystematic risks, $\left|\Delta_k/\sqrt{\Lambda_k}\right|$. We calibrate this variable based on the empirical evidence provided by Calvet, Campbell, and Sodini (2007) using the portfolio returns of Swedish households. They do a decomposition of the variance of portfolio returns and find that—for the household with the median total risk—more than half of the variance (more precisely, 54.9%) is explained by idiosyncratic risks as opposed to systematic risks. We take the speculative Sharpe ratio to be equal to the Sharpe ratio on the market portfolio, $\left|\Delta_k/\sqrt{\Lambda_k}\right| = 0.4$. This ensures that, when the market is complete, half of the portfolio variance for the investors in our model is driven by idiosyncratic risk. This can be viewed as a conservative calibration since the markets were arguably not fully complete over the time period (1999-2002) in which Calvet, Campbell, and Sodini (2007) conduct their study.

Figure 3 illustrates how customization affects the asset returns with this calibration. The x-axes correspond to the degree of customization, $\bar{i}_A/(K-1)$, which we vary over the range, $[0,1]$. The top panel shows that increasing customization reduces the risk-free rate and leaves the risk premium on the market portfolio constant, consistent with Proposition 2. The dashed lines illustrate the solution with $\varepsilon = 1$, which provides a useful comparison by shutting down the choice channel. Note that (for $\varepsilon = 2$) the effects are quantitatively sizeable. Going from zero customization to full customization reduces the risk-free rate by almost 1 percentage point.

5.2 Customization in More General Settings

We next discuss the extent to which the customization result generalizes beyond our benchmark model.

Disagreement on the Market Portfolio Recall that we rule out disagreement on the market portfolio by assuming (15). Absent this assumption, the effect of customization is largely unchanged in our numerical simulations, even though we are unable to prove an

equity premium puzzle (Bansal and Yaron, 2004).

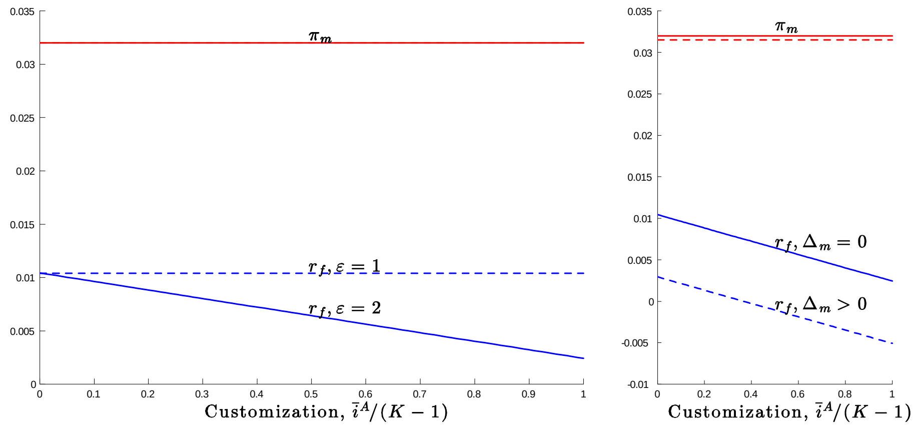

Figure 3: The left panel illustrates the effect of increased customization on asset returns for the benchmark calibration $\varepsilon = 2$ (solid lines), and the comparison case $\varepsilon = 1$ (dashed lines). The right panel illustrates the asset returns without (solid lines) and with (dashed lines) disagreement on the market portfolio.

analytical result. To see this, consider the example plotted in Figure 3 with the only difference that investors also disagree about the market portfolio. Specifically, an investor who is optimistic (resp. pessimistic) about a nonsystematic risk $k < K$ is also optimistic (resp. pessimistic) about the systematic risk, $\log \varphi_m = z_K$. We calibrate the level of disagreement on the market portfolio by assuming, $\left|\Delta_m/\sqrt{\Lambda_m}\right| = \left|\Delta_k/\sqrt{\Lambda_k}\right|$, so that the speculative Sharpe ratios on systematic and nonsystematic risk factors are the same. The right panel of Figure 3 illustrates the results of increased customization in this case. Compared to the earlier case with $\Delta_m = 0$, the risk-free rate is uniformly lower. The risk premium is also slightly lower, but the difference is not discernible. More importantly, increased customization reduces the risk-free rate and does not have a discernible effect on the risk premium, as in Proposition 2, even though condition (15) is violated.

Absent condition (15), investors take speculative positions on the market portfolio as well as the nonsystematic risk factors. This generates an additional increase in their certainty-equivalent returns, and reduces the risk-free rate further, as illustrated by Figure 3. However, speculation on the market portfolio also breaks the symmetry between optimists' and pessimists' returns in Eq. (18). Since the asset m is in positive supply, all investors are its natural buyers. Even if optimists did not adjust their positions (relative to the average investor), their perceived return would be higher simply because they are already holding the market portfolio. Therefore, in equilibrium, optimists obtain a greater certainty-equivalent return—and hold more assets—relative to pessimists. This asymmetry implies that belief disagreements can potentially also affect relative asset prices and risk premia, which makes a theoretical characterization difficult. However, for empirically relevant parameters, these asymmetric effects are very small, as illustrated by Figure 3, and the effect of greater customization remains qualitatively unchanged.

Short-selling Constraints In our model, we assume the investors can short sell the risky assets without constraints. When short-selling constraints bind on some assets, there are additional complications but the effect of customization remains qualitatively unchanged.[15] We establish this formally in Appendix A.3. There, we assume the investors cannot short sell a fraction of the nonmarket assets , $\tilde{\mathbf{J}} \subset \{1, .., K-1\}$. We continue to make all of the other assumptions in Lemma 1 (including no disagreement on the market portfolio). We also assume $n^{i_A} > 0$ for each $i_A \in I_A$, that is, there is a positive mass of investors of each access type (even before customization improves the market access).

Under these assumptions, the appendix characterizes the risk premia as,

$$\pi_j = \begin{cases} \frac{\Lambda_{jm}}{\Lambda_m}\pi_m & \text{for } j \notin \mathbf{J} \setminus \tilde{\mathbf{J}} \\ \frac{\Lambda_{jm}}{\Lambda_m}\pi_m - \max_{(\mathbf{i}_A, \mathbf{i_B})} \Delta_j^{(\mathbf{i}_A, \mathbf{i_B})} & \text{for } j \in \tilde{\mathbf{J}} \end{cases}, \text{ where } \pi_m = \gamma \Lambda_m. \tag{19}$$

Here, $\Delta_j^{(\mathbf{i}_A, \mathbf{i_B})}$ (defined in the appendix, by equation A.3) represents an individual's excess valuation of the asset relative to the average investor. This term depends on the investor's optimism regarding asset j's payoff plus its value as a hedge for other speculative positions that the agent takes in equilibrium. Hence, compared to the characterization in Lemma 1, the (log) relative price of an asset that cannot be short sold is increased by the maximum of the term, $\Delta_j^{(\mathbf{i}_A, \mathbf{i_B})}$, across agents. Intuitively, these assets are now priced by their highest valuation investors, as first emphasized by Miller (1977).

The appendix also shows that the risk-free rate is found by solving Eq. (16) as before, but the certainty-equivalent return is now given by,

$$r_{ce}^{\left(\bar{i}_A, \mathbf{i}_B\right)} = r_f + \frac{\pi_m^2}{2\gamma\Lambda_m} + \frac{1}{2\gamma}\left(\mathbf{F}'_{J^{i_A}\setminus\tilde{\mathbf{J}}}\left(\mathbf{i}_B\right)\right)' \Lambda^{-1}_{J^{i_A}\setminus\tilde{\mathbf{J}}}\left(\mathbf{F}'_{J^{i_A}\setminus\tilde{\mathbf{J}}}\left(\mathbf{i}_B\right)\right). \tag{20}$$

Comparing this expression with Eq. (17), the main difference is that the assets with short-selling constraints are effectively not traded.

Using this characterization, Proposition 6 shows that greater customization reduces the risk-free rate and leaves the risk premia unchanged—generalizing Proposition 2. Intuitively,

[15] We should note that our modeling strategy makes short selling seem more relevant than it would be in practice. For tractability, we assume there is a single asset m in positive net supply, and nonmarket assets $j \neq m$ are in zero net supply. Thus, an investor who would like to reduce her exposure to a nonmarket asset is required to short sell. In practice, most nonmarket assets (such as stocks or bonds) would be in positive net supply. An investor who is pessimistic about these assets could simply not include them in her portfolio. The short-selling constraints would start to bind only if the investor is substantially pessimistic.

for assets on which the short-selling constraints do not bind, customization has the same qualitative effect as before. One could expect customization to have different effects on constrained assets, but that turns out not to be the case either. For those assets, there are additional relative price effects, as illustrated by (19), but these effects are there both before and after financial innovation that expands customization. Thus, short-selling constraints dampen speculation and mitigate the quantitative effects of greater customization, as illustrated by Eq. (20), but they leave the qualitative effects unchanged.

Investment We have so far assumed that the assets are in fixed supply. The effect of customization remains qualitatively unchanged if we allow for investment and endogenize the supply of the market portfolio. We establish this formally in Appendix A.4. We assume that the output of the economy at time 1 is produced by capital and labor according to the function, $\Phi(\mathbf{z})\, G(K, L)$. Here, $G(K, L)$ denotes a neoclassical aggregate production function and $\Phi(\mathbf{z})$ denotes a Hicks-neutral productivity shock which satisfies, $\log \Phi(\mathbf{z}) = \mathbf{F}_m' \mathbf{z}$. Investors in this economy are also workers and supply one unit of labor inelastically. Capital is produced at time 0 by a competitive sector of investment goods firms that can convert one unit of the consumption good at time 0 to one unit of capital at time 1. The capital depreciates fully after use so that the capital stock at date 1—which is equal to the supply of the market portfolio—is determined by new investment. We maintain the same structure of investors' market access and beliefs as in the baseline setting.

In this setting, Proposition 7 in the appendix shows that greater customization reduces the risk-free rate and leaves the risk premia unchanged—generalizing Proposition 2—while also increasing investment. Intuitively, the risk-free rate declines for the same reason as in the benchmark analysis. In turn, the decline in required asset returns stimulates aggregate investment. The analysis in the appendix also shows that the investment response dampens the effect on asset returns. Hence, in this setting, the quantitative impact on the risk-free rate also depends on the properties of the aggregate production function (specifically, the elasticity of substitution between capital and labor).

6 Market Participation

Our analysis so far has focused on increased customization, assuming that the investors already have access to the market portfolio. However, financial innovation in postwar years has also increased investors' access to the market portfolio. Figure 4 shows that stock market participation in the US has rapidly increased from 1950s until late 1990s. In this section, we analyze how greater choice that comes in the form of participation affects asset returns. We

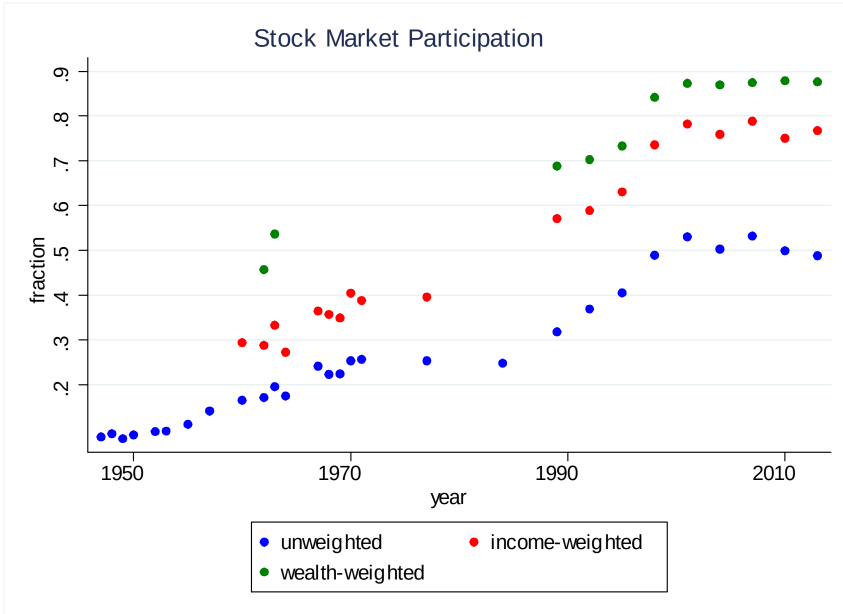

Figure 4: The figure shows the fraction of households in the US that invest in stocks over the period 1947-2013. The plots are based on the authors' calculations using data from the Michigan Survey of Consumer Finances (1947-1977), the PSID (1984), and the Survey of Consumer Finances (1989-2013).

show that greater participation tends to reduce the return on risky assets but it typically increases the risk-free rate, in contrast to greater customization.

We capture market participation with improved access to asset m. In particular suppose there are two types of investors, $i \in \{1, 0\}$, with market access sets respectively given by $J^1 = \{m\}$ and $J^0 = \emptyset$. Type 1 investors have access to asset m, in addition to the risk-free asset. In contrast, type 0 investors have access to only the risk-free asset. The relative mass of participants, n^1, captures the degree of participation. To simplify the exposition, we also assume investors are identical in all other dimensions. In particular, they share the same beliefs, $\boldsymbol{\mu}^i_{\mathbf{z}} = \boldsymbol{\mu}_{\mathbf{z}}$, which implies $\mu^i_m = \mu_m$, for each i.[16] Investors also have the same preference parameters, $\beta^i = \beta, \varepsilon^i = \varepsilon$, and $\gamma^i = \gamma$, which implies, $a^i(\cdot) = a(\cdot)$, for each i. The following lemma characterizes the equilibrium.

Lemma 2. *Consider the above setup with common beliefs and limited participation in the*

[16]This assumption ensures that the market portfolio m is sufficient to construct an efficient portfolio for each investor. Thus, allowing further access to assets $j \in \{1, .., K-1\}$ would not change the equilibrium in this section.

market portfolio. There exists a unique equilibrium in which π_m *and* r_f *jointly solve,*

$$\pi_m = \gamma\Lambda_m \left(1 + \frac{1-n^1}{n^1}\frac{a(r_f)}{a(r_{ce}^1)}\right), \tag{21}$$

$$\frac{\eta_m P_m}{Y_0 + \eta_m P_m} = \left(1-n^1\right) a(r_f) + n^1 a\left(r_{ce}^1\right), \tag{22}$$

where $P_m = \exp\left(\mu_m + \frac{\Lambda_m}{2} - r_f - \pi_m\right)$*, and*

$$r_{ce}^1 = r_f + \frac{\pi_m^2}{2\gamma\Lambda_m}. \tag{23}$$

To understand the result, note that the market risk premium with full participation would be given by $\pi_m = \gamma\Lambda_m$ (see Lemma 1). Eq. (21) shows that the risk premium with limited participation is greater, and more so if the degree of participation is lower. Intuitively, the aggregate risk is shared among fewer investors—the participants—which require a greater premium as compensation for holding additional risks. Eq. (22) is a market clearing condition for all assets as before. Eq. (23) characterizes the certainty-equivalent return for investors that have access to the market portfolio. In view of the choice channel, market access increases the certainty-equivalent return, $r_{ce}^1 > r_f$ (by enabling the investors to earn the aggregate risk premium). The next result describes how increasing the relative mass of participation, $n^1 \in [0,1]$, affects asset returns.

Proposition 3 (Increased Participation)**.** *Consider the equilibrium characterized in Lemma 2. Financial innovation that increases the relative mass of participants,* n_1*, decreases the risk premium,* π_m*, and decreases the expected return on the market portfolio,* $r_f + \pi_m$*.*

The effect on the risk-free rate r_f is theoretically ambiguous. In our numerical analysis, we also find that greater participation also typically increases the risk-free rate, r_f.

The result on the return on the market portfolio follows from the choice channel. Greater access to the market portfolio increases the average demand for assets, as captured by Eq. (22), which increases the valuation of all assets. The result on the risk premium follows from improved aggregate risk sharing. Unlike greater customization, greater participation makes risky assets relatively more valuable as captured by Eq. (21). This also explains why the result on the risk-free rate is theoretically ambiguous. The absolute price effect tends to reduce the risk-free rate as before, but the relative price effect tends to raise it. We next numerically illustrate these results and gauge the net effect on the risk-free rate.

Numerical Illustration We use the same parameters from Section 5.1: in particular, $\gamma = 5, \varepsilon = 2$, and $\sqrt{\Lambda_m} = 8\%$—which leads to $\pi_m/\sqrt{\Lambda_m} = 0.4$ with full participation.

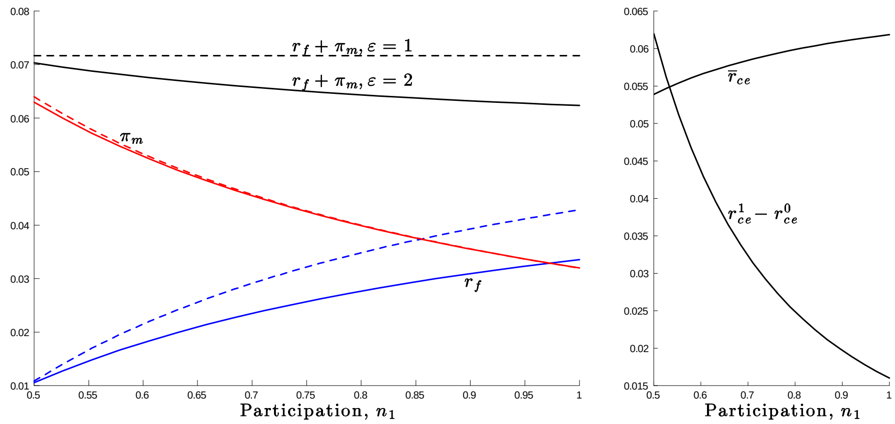

Figure 5: The left panel illustrates the effect of increased customization on asset returns for the benchmark calibration $\varepsilon = 2$ (solid lines), and the comparison case $\varepsilon = 1$ (dashed lines). The right panel illustrates the return-gain from participation, as well as the avreage certainty-equivalent return.

Motivated by Figure 4 (the wealth-weighted measure), we investigate the effect of varying the degree of participation, n_1, from 50% to 100%.

The left panel of Figure 5 illustrates the results of this exercise. The solid lines show that increased participation reduces the risk premium and the expected return on the market portfolio, consistent with Proposition 3, while also increasing the risk-free rate. The dashed lines illustrate the solution with $\varepsilon = 1$, which provides a useful comparison case. In this case, the relative price effects are still active but the absolute price effects are absent since the choice channel is shut down. Comparing this case with our calibration, $\varepsilon = 2$, shows that the choice channel from increased participation reduces the asset returns by about 1 pp. However, this effect is too small to overturn the relative price effect, which is about 3 pp. On net, greater participation increases the risk-free rate by about 2 pp—in contrast to greater customization (cf. Figure 3).

The choice channel is weakened in this context because of the crowd-out effects that tend to lower the (marginal) benefit from participation. Greater n_1 implies a smaller risk premium, and thus a smaller certainty-equivalent return for investors that already participate [cf. Eq. (23)]. These investors react by reducing their asset holdings, which mitigates the effect of increased choice on asset prices. The right panel of Figure 5 illustrates this crowd-out effect by plotting the gains from participation, $r^1_{ce} - r^0_{ce}$. The panel also plots the average certainty-equivalent return, $\overline{r}_{ce}$, defined as the solution to $a(\overline{r}_{ce}) = (1 - n^1)\, a(r^0_{ce}) + n^1 a(r^1_{ce})$. For $n_1 = 0.5$, participants earn about 6pp greater certainty-equivalent return than nonparticipants, but this difference falls to less than 2pp for greater levels of participation. Consequently, the average return, $\overline{r}_{ce}$, increases by a relatively small amount.

These crowd out effects are absent from our analysis of greater customization in the previous section. There, optimists and pessimists that speculate on an asset do not change the relative price of that asset. Thus, they do not preclude other investors from taking speculative positions on other (or even the same) assets. With customization, the quantitative strength of the choice channel depends on the size of disagreements (as well as ε) but it is not dampened by endogenous forces. Hence, while the choice channel can also apply when investors have homogeneous beliefs—as illustrated by our analysis in this section—its general equilibrium effects are more powerful when investors have heterogeneous beliefs, and when they gain access to assets that enable them to speculate on their beliefs.

7 Demand for Safety and Securitization

Another important innovation to have expanded investors' choice in recent years is structured finance or securitization. Securitization converts risky or illiquid assets into marketable securities. Nontraditional forms of securitization further distribute the cash flows according to investors' risk preferences or beliefs. The growth of nontraditional securitization is often linked with another phenomenon: the growing demand for safe assets from the governments of fast-growing emerging markets such as China as well as other sources (see Gennaioli, Shleifer, and Vishny, 2012). Figure 6 shows that the rapid growth of nontraditional securitization in early 2000s has coincided with the growth of safe-asset holdings by emerging market central banks. In this section, we analyze how securitization that endogenously comes about to meet investors' demand for safety affects aggregate savings and asset returns.

We analyze these issues using a variant of the setup in the previous section. Suppose there are two types of investors, denoted by $\{1, 2\}$, that have common beliefs (so it suffices to restrict attention to access to the market portfolio, m). Type 2 investors participate only in the safe asset, $J^2 = \emptyset$, similar to type 0 investors before. We now interpret these investors as corresponding to a special group of agents—which we will refer to as emerging market (EM) governments for concreteness—that have a preference for safe assets in addition to having a high demand for assets. To capture the latter feature, we allow these investors to have different preference parameters, β^2, ε^2, that induce a different asset holdings function, $a^2(\cdot)$. We consider parameters that ensure these investors hold more assets than the other investors in equilibrium.

To model securitization, we also depart from our earlier analysis by assuming that the type 1 investors face an additional constraint that prevents them from borrowing via the risk-free asset. Securitization is an intermediation technology that relaxes this constraint.

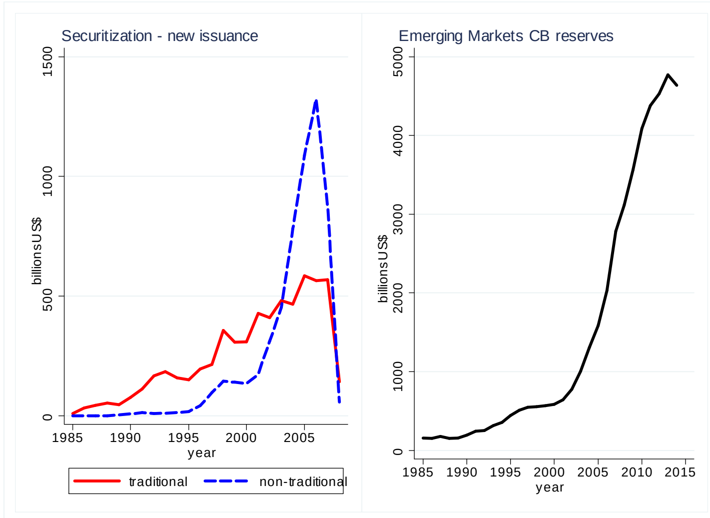

Figure 6: The left panel is from Chernenko, Hanson, and Sunderam (2014), based on data from the Securities Data Company. Traditional securitizations include commercial mortgage backed securities (MBS), prime residential MBS, and asset backed securities. Nontraditional securitizations include nonprime residential MBS and collateral debt obligations. The right panel illustrates the evolution of the reserve assets held by the central banks in emerging markets (source: the World Bank). Amounts are in constant year 2000 US dollars.

Specifically, type 1 investors have access to all (relevant) risky assets, $J^1 = \{m\}$, and they an choose from one of two investment options. They can invest directly, in which they face an additional constraint, $\omega_f^1 \geq 0$, that prevents them from short-selling the risk-free asset. Alternatively, they can invest via an intermediary that is able to securitize the risky assets. The intermediary is not subject to the borrowing constraint, but it also charges a competitive fee that captures (unmodeled) cost of securitization. For simplicity, the fee reduces the certainty-equivalent return on assets by a constant amount denoted by $c > 0$.

In the appendix, we calculate the net certainty-equivalent return from direct and securitized investment respectively as,

$$r_{ce}^{(1,-)} = r_f + \pi_m - \frac{\gamma}{2}\Lambda_m \text{ and } r_{ce}^{(1,+)} = r_f + \frac{\pi_m^2}{2\gamma\Lambda_m} - c. \tag{24}$$

For the relevant range of the risk premium ($\pi_m \geq \gamma\Lambda_m$), the gross (pre-fee) return on securitized investment exceeds the return on direct investment, $r_{ce}^{(1,+)} + c \geq r_{ce}^{(1,-)}$. Moreover, the difference increases in the risk premium, which illustrates that securitization enables investors to make a leveraged investment in risky assets. We assume that type 1 investors can freely choose between direct investment and securitization, which implies $r_{ce}^1 = \max\left(r_{ce}^{(1,-)}, r_{ce}^{(1,+)}\right)$. We let $n^+ \in [0,1]$ denote the relative mass of type 1 investors that choose the securitization option in equilibrium. The rest of the equilibrium is defined as before.

In a benchmark without EM government savings, $n^2 = 0$, the risk premium is given by $\pi_m = \gamma \Lambda_m$ as in the previous section and there is no securitization, $n^+ = 0$. When $n^2 > 0$, there is some securitization in equilibrium. Moreover, securitization takes an interior value, $n^+ \in (0,1)$, if and only if the net return from direct investment and securitization are equated, $r_{ce}^{(1,-)} = r_{ce}^{(1,+)}$. Using (24), there is a threshold level of the risk premium that satisfies this indifference condition. We denote the solution by $\bar{\pi}_m(c)$, and note that it satisfies $\bar{\pi}_m(c) > \gamma \Lambda_m$: that is, the EM government savings increases the risk premium beyond its benchmark level. Intuitively, a greater risk premium is necessary to incentivize securitization and to meet the excess demand for safe assets. Lemma 5 in the appendix characterizes the rest of the equilibrium. The following result establishes its comparative statics.

Proposition 4 (EM Savings and Endogenous Securitization)**.** *Suppose $\epsilon^2 \leq \epsilon^1, n^2 > 0$, and consider an equilibrium characterized in Lemma 5 that features $n^+ \in (0,1)$ and $a^2(r_f) \geq a^1(r_{ce}^1)$.*

(i) An increase in the relative mass of type 2 investors, n^2, decreases the risk-free rate, r_f , increases securitization, n^+, and leaves unchanged the risk premium at $\bar{\pi}_m(c) > \gamma \Lambda_m$.

(ii) A decrease in the cost of securitization, c, increases n^+, increases r_f, and decreases $\pi_m = \bar{\pi}_m(c)$ towards $\gamma \Lambda_m$.

The first part shows that increasing EM governments' relative wealth share decreases the risk-free rate, consistent with the savings glut hypothesis (see, for instance, Bernanke, 2005). The EM governments have high desired savings and a preference for safe assets: both channels tend to reduce r_f. The result also shows that greater n^2 endogenously leads to a greater degree of securitization, n^+. The intuition is that the EM governments' savings tends to increase the risk premium, π_m. This makes securitization more attractive, as captured by Eq. (24), which in turn increases n^+. Once the adjustment is complete, π_m remains unchanged but the degree of securitization is greater.

The second part implies that greater securitization—driven by a reduction in the costs of securitization—reduces the risk premium and increases the interest rate. Intuitively, securitization—just like participation—improves aggregate risk sharing and increases the relative price of risky assets. Combining this result with the first part, we also obtain that greater n^2 decreases r_f relatively less when the degree of securitization is greater. Intuitively, securitization dampens the strong relative price effects of the EM government savings.

We next illustrate these results with a numerical calibration, which is also useful to investigate type 1 investors' savings. Under our interpretation of type 2 investors as EM governments, type 1 investors' savings can be thought of as the model analogue of the current accounts in developed countries. We vary the EM investors' wealth over some range, $n^2 \in [0, \bar{n}^2]$, so as to capture the increase in EM government savings between the late 1990s and 2006 (before the crisis). We investigate how this change affects asset returns and current accounts.

We use the baseline parameters as in the earlier sections: in particular, $\gamma = 5, \sqrt{\Lambda_m} = 8\%, \varepsilon^1 = 2$, and β^1 chosen so that $r_f = 1\%$ when $n^2 = 0$. We calibrate the cost of securitization, c, so that positive EM government savings $n^2 > 0$, increases the market risk premium by 1pp relative to the baseline, $\bar{\pi}_m(c) - \gamma\Lambda_m = 1\%$. This increase is roughly consistent with the 2-3pp increase in the equity risk premium over the calibration period (1999-2006) illustrated in Figure 1.

The parameters that are harder to calibrate concern the type 2 investors' preferences, ε^2 and β^2, and their mass, $\bar{n}^2$. As a benchmark, we set $\varepsilon^2 = \varepsilon^1 = 2$ so type 2 agents' desired asset holdings is equally reactive to asset returns as type 1 agents. Since β^2 and n^2 affect the equilibrium in a similar way, we set β^2 at a relatively high level ($\beta^2 = 2\beta^1$) and check robustness to other choices.[17] We then calibrate $\bar{n}_2$ so that increasing the EM savings from 0 to $\bar{n}^2$ in the model decreases type 1 agents' savings by 4pp—which is roughly consistent with the 4pp decline in the US current account deficit over the calibration period. Note that we do not match the risk-free rate in any of our calibrations: that is, we let the model speak about the quantitative impact on r_f.

Figure 7 illustrates the results from this exercise. The solid lines plot our benchmark calibration, whereas the dashed lines plot a comparison case with considerably higher cost of securitization ($\tilde{c} = 10c$). The difference between the solid and the dashed lines describe the effect of securitization. The top left panel illustrates how increasing n_2 endogenously induces more securitization. The bottom left panel shows that the increase in the wealth of EM governments increases the risk premium by 1pp (by assumption) and decreases the risk-free rate by 3pp. Hence, the effect on r_f is quite sizeable. The figure also illustrates that the effect on r_f (as well as π_m) would have been even greater in absence of securitization.

The top right panel illustrates the effect on investors' certainty-equivalent returns. The return declines for both types of investors in view of the decline in r_f, but the decline is dampened for type 1 agents in view of the increase in π_m. The figure also reveals a key insight from our analysis that securitization increases the certainty-equivalent return for type 2 investors. Absent securitization, the risk-free rate would have fallen more and type 2 investors would have earned lower returns. In contrast, securitization does not have much

[17] As long as $\beta^2 > \beta^1$, increasing type 2 investors' savings (via greater β^2) or their relative mass (via greater n^2) have a similar effect on aggregate asset demand.

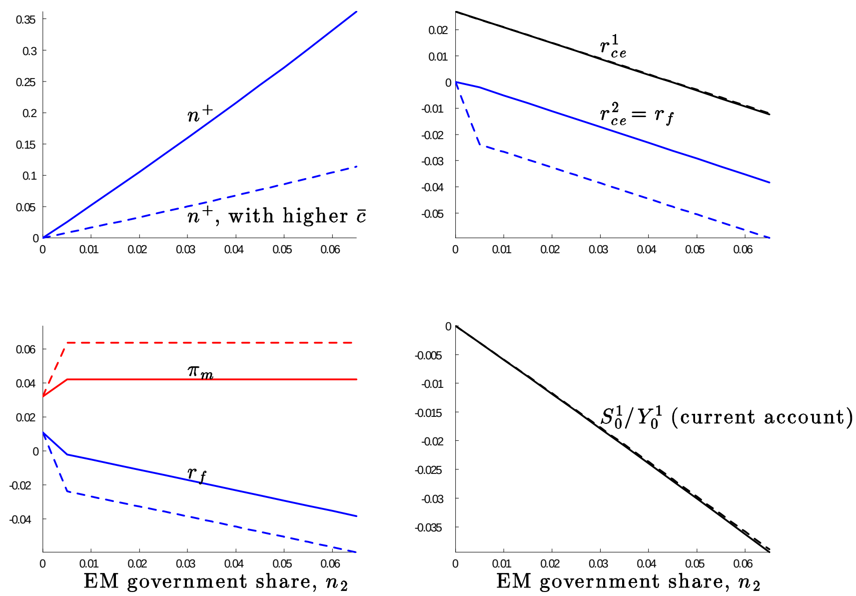

Figure 7: The plots illustrate the effect of increasing the emerging market share, n_2, on various equilibrium variables. The solid lines correspond to the benchmark calibration, and the dashed lines correspond to a comparison case with higher marginal cost of securitization.

of an effect on type 1 investors' returns: for them, the greater increase in the risk premium would have roughly countered the greater decline in the risk-free rate.

The bottom-right panel illustrates the effect on type 1 investors' savings rate, S^1_0/Y^1_0. High savings by EM governments translate into low equilibrium savings by type 1 investors—pushing them into current-account deficits. Our numerical analysis also reveals that securitization *exacerbates* these current-account deficits—although the effect is small with our calibration and not very visible in the figure. In particular, absent the securitization response, the savings of type 1 investors would have been greater. Intuitively, since securitization induces type 1 investors to earn greater returns, it also induces them to save more in equilibrium (relative to the counterfactual)—a manifestation of the choice channel in this context. The effect is not large because type 2 investors have a relatively small mass and they are not too reactive to equilibrium returns since we assume $\varepsilon^2 = 2$. The extent to which the EM governments' savings would react to a change in the risk-free rate is an empirical question, which we leave for further research.

8 Conclusion

Rapid financial innovation in recent years has vastly expanded the portfolio choice of investors. We theoretically investigate the implications of financial innovation for investors' savings and asset returns. Our main result establishes a choice channel by which, under mild assumptions, an investor that gains access to greater portfolio choice increases her savings. In equilibrium, greater savings exert a generally downward pressure on asset returns, but the precise effects depend on the type of financial innovation. We show that greater portfolio customization reduces the expected return on all assets, including the risk-free rate, without affecting the risk premia. This result is quantitatively significant and qualitatively robust to the presence of various types of disagreements or various frictions (such as short-selling constraints). We also find that, for empirically relevant parameters, greater participation increases the risk-free interest rate and reduces the risk premium.

We further extend our model to analyze the effect of securitization in an environment with growing savings by investors that seek safety—such as the emerging market governments. These investors' savings reduces the risk-free rate and increases the risk premium. Securitization dampens these investors' pricing effects by meeting their choice for safety, which in turn induces them to save more and exacerbates the global savings imbalances.

While our analysis is theoretical, our results are broadly consistent with various trends in financial innovation and asset returns in developed economies over the last half century. The increase in market participation between 1950 and the early 1980s might have contributed to the decrease in the risk premium and the increase in the risk-free rate over this period. More subtly, the proliferation of portfolio customization starting in the early 1980s might have contributed to the secular decline of the risk-free rate in recent decades. Securitization might have increased in response to growing demand for safety in late 1990s. Its collapse after the recent financial crisis might have contributed to the sharp reduction in the risk-free interest rate, the increase in the risk premium, as well as the reduction in the US current account deficit in recent years. These trends have undoubtedly many other contributing factors: our point is that financial innovation and the choice channel are among those contributors.

Our results also shed some light on the low interest rates in recent years. These rates are worrisome from a macroeconomic policy point of view, as they increase the likelihood of liquidity trap episodes in which monetary policy is constrained. Our analysis suggests that financial innovation that facilitates portfolio customization might be a contributing factor to low interest rates. We also show that other types of financial innovation that facilitate participation or securitization might help to increase the interest rates.

Even though our analysis has been purely positive, our model also has policy implications.

For instance, through the lens of our model, restricting portfolio customization or subsidizing securitization might be beneficial by reducing the incidence of liquidity traps. More generally, our analysis highlights that financial innovation affects investors' consumption and savings decisions, with implications for aggregate demand. Economic agents that introduce or adopt these financial innovations do not internalize their effects on aggregate demand, which might create inefficiencies (see Korinek and Simsek, 2014). We leave a more complete analysis of the interaction between financial innovation and aggregate demand externalities for future work.

A Appendix A: Omitted Extensions

A.1 Precautionary Channel

In the main text, we focused on the cases in which investors effectively do not face any background risks so that they do not have precautionary savings concerns. We next illustrate the effect of financial innovation in an environment with precautionary savings. We obtain an alternative precautionary channel of financial innovation, and contrast it with our choice channel.

Consider the setup in Section 3. Suppose, in addition, that there is a risk-neutral belief distribution, $\{q^n(\mathbf{z})\}_Z$, that prices all assets, that is,

$$P_j = \frac{P_f}{\overline{\varphi}_f} \sum_{\mathbf{z} \in Z} q^n(\mathbf{z}) \varphi_j(\mathbf{z}) \text{ for each } j \in \mathbf{J}. \tag{A.1}$$

This assumption holds, for instance, if there is a positive mass of investors that can access all assets (which implies no arbitrage). Suppose also that the investor's belief satisfies the following.

Assumption 1$^{\text{P}}$. $q^i(\mathbf{z}) = q^n(\mathbf{z})$ for each $\mathbf{z} \in Z$, which also implies $E^i[\cdot] = E^n[\cdot]$.

This assumption is quite strong, as it implies that the investor's (perceived) expected return on every asset is the risk-free interest rate. Under this assumption, a risk-averse investor without background risks would not want to hold any risky financial assets. Hence, the assumption ensures that the investor demands financial assets only to hedge her background risks.

Proposition 5 (Precautionary Channel). *Suppose there is a risk-neutral distribution [cf. (A.1)], Assumption 1^P holds, and the investor has CRRA preferences, $\gamma^i = 1/\varepsilon^i$. Suppose also that financial innovation completes the market in the sense that, for each $\mathbf{z} \in Z$, there exists $j_{\mathbf{z}} \in J^{i,new}$ such that $\varphi_{j_{\mathbf{z}}}(\mathbf{z}) > 0$ and $\varphi_{j_{\mathbf{z}}}(\tilde{\mathbf{z}}) = 0$ for each $\tilde{\mathbf{z}} \neq \mathbf{z}$. Then, financial innovation reduces the investor's asset holdings (and thus, savings), $A_0^{i,new} \leq A_0^{i,old}$, with strict inequality if $C_1^{i,old}(z_1) \neq C_1^{i,old}(z_2)$ for some $z_1, z_2 \in Z$ and $A_0^{i,old} > A^{i,\min}$.*

Hence, consistent with much of the precautionary savings literature (see Section 1.1), financial innovation induces the investor to save less, and strictly so when she faces some income risks before innovation. Intuitively, when markets are incomplete, the investor saves for precautionary reasons. This is because she faces some background risks, and the time-separable CRRA preferences satisfy the prudence condition. Financial innovation enables the investor to hedge her risks. By doing so, it alleviates the precautionary demand for saving, thereby reducing savings.

This intuition also illustrates the fragility of the result. The argument relies on the fact that the investor demands the new financial assets mainly to reduce her portfolio risks. If instead the new financial assets increase the investor's portfolio risks, perhaps because they enable her to participate in sharing the aggregate risk or speculate against other investors, then the argument does not hold. In fact, in the main text, we have argued that the opposite result holds under mild assumptions. Specifically, Proposition 1 in the main text drops Assumption 1^{P} and replaces it with the assumption that investors do not face any background risks. This means that the investor holds new risky assets only to share aggregate risks or to speculate against other investors. In this alternative setup, as long as $\varepsilon^i > 1$, providing the investor with greater choice induces her to save more, in sharp contrast to Proposition 5.

A.2 Choice Channel Comparative Statics

The effect of financial innovation on savings depends on the increase in the investor's certainty equivalent return, R^i_{ce}, as well as the extent to which the investor reacts to this increase. Our next result formally establishes the comparative statics for the saving rate, $S^i_0/Y^i_0 = \left(A^i_0 - \sum_j x^i_{-1,j} P_j\right)/Y^i_0$.

Observation 1. *The increase in the saving rate,* $\left(S^{i,new}_0 - S^{i,old}_0\right)/Y^i_0$*, is*

(i) increasing in the value of financial assets, W^i_0*,*

(ii) If $J^{old} = \{f\}$*, then decreasing in risk aversion,* γ^i*.*

The first part generates the testable implication that the saving rate increases relatively more for wealthier investors. This result follows from two channels that operate in the same direction. First, all else equal, an investor with lower wealth is more likely to face a binding borrowing constraint. When the constraint binds, the choice channel might affect the investor's portfolio but it has a smaller impact (often, no impact) on her savings. These investors' saving rate is primarily determined by the constraint, which we take as exogenous and fixed. In fact, if we were to let the lower bound, $A^{i,\min}$, decline alongside with the expansion of choice (e.g., to capture other financial innovations that might have relaxed borrowing constraints), then the saving rates of the constraint investors would decline. In contrast, a wealthy investor is less likely to face a borrowing constraint. When the constraint does not bind, the investor increases her savings in response to greater choice—even if $A^{i,\min}$ declines.

There is a second reason for why greater wealth is associated with a stronger choice channel. Solving the investor's portfolio problem reveals that greater choice induces a multiplicative increase in the investor's desired asset holdings, $\tilde{A}^i_0$ (see Eq. (12)). For instance, greater choice might increase the investor's desired asset holdings by 1%. This type of change generates a greater dollar increase in the assets (and ultimately savings) of wealthier investors, because they start from a greater asset base, $A^{i,old}_0$.

The second part says that greater choice increases savings relatively more for less risk averse investors. While we do not directly observe risk aversion, this is likely to be reflected in the amount of risk in investors' portfolios. Hence, this part generates the additional testable prediction that the choice channel is stronger for investors whose (pre-innovation) portfolios are riskier. The intuition follows from the observation that the choice channel concerns improvements in investors' access to risky assets. Investors that are less risk averse, and therefore more willing to hold risky assets, naturally react more to an expansion of risky choice. The formal result requires the additional condition that the initial choice set consists only of the risk-free asset. However, the result applies more generally—regardless of the initial choice set—once we impose more structure on asset payoffs and convert the portfolio problem into mean-variance optimization (see Eq. (11) in Section 4).

A.3 Customization with Short Selling Constraints

In the main text, we abstracted away from short-selling constraints. In this section, we generalize our benchmark result on customization in Section 5 to a setting with short-selling constraints. Suppose the investors cannot short sell a fraction of the nonmarket assets , $\tilde{\mathbf{J}} \subset \{1, .., K-1\}$. Formally, the portfolio problem (10) features an additional constraint,

$$\omega_j^i \geq 0 \text{ for each } j \in \tilde{\mathbf{J}}. \tag{A.2}$$

We continue to make all of the other assumptions in Lemma 1 (including no disagreement on the market portfolio). We also assume $n^{i_A} > 0$ for each $i_A \in I_A$, that is, there is a positive mass of investors of each access type (even before customization improves the market access).

The following lemma characterizes the equilibrium with short-selling constraints. To state the result, we define the notation $\Lambda_{J^{i_A}} = \begin{bmatrix} \Lambda_{\tilde{J}^{i_A}} & \tilde{\Lambda}_{\tilde{J}^{i_A}} \\ \left(\tilde{\Lambda}_{\tilde{J}^{i_A}}\right)' & \Lambda_{J^{i_A}\backslash\tilde{\mathbf{J}}} \end{bmatrix}$ for any investor with market access i_A, where $\tilde{J}^{i_A} = J^{i_A} \cap \tilde{\mathbf{J}}$. We also let $\tilde{\Lambda}_{j,\tilde{J}^{i_A}}$ denote the j-th row of $\tilde{\Lambda}_{\tilde{J}^{i_A}}$.

Lemma 3. *Consider the setting in Lemma 1 with the short-selling constraints in* (A.2). *Then, there exists an equilibrium in which:*

(i) The risk premia satisfy (19) *in the main text where*

$$\Delta_j^{(\mathbf{i}_A,\mathbf{i_B})} \equiv \mathbf{F}_j'\mathbf{i_B} - \tilde{\Lambda}_{j,\tilde{J}^{i_A}} \Lambda^{-1}_{J^{i_A}\backslash\tilde{\mathbf{J}}} \left(\mathbf{F}_{J^{i_A}\backslash\tilde{\mathbf{J}}}\right)' \mathbf{i_B}. \tag{A.3}$$

(ii) The risk-free rate is the unique solution to Eq. (16)*, where* $r_{ce}^{(\bar{i}_A,\mathbf{i}_B)}$ *satisfies* (20) *in the main text.*

Here, $\Delta_j^{(\mathbf{i}_A,\mathbf{i_B})}$ captures an investor's excess valuation of the asset relative to the average investor. The first part says that the asset is now priced by the investor that has the highest valuation. Hence, short-selling constraints change the characterization of relative asset prices. However, they leave the characterization of the risk-free rate largely unchanged. In particular, the second part says that the risk-free rate is determined by Eq. (16) as before. The difference is that the investors' certainty-equivalent returns are determined as if the assets on which the short-selling constraints bind are not available for trade (see (20)). This is intuitively because short-selling constraints dampen speculation.

Lemma 3 leads to the following generalization of our main result on customization, Proposition 2.

Proposition 6 (Customization with Short Selling Constraints)**.** *Consider the setting in Lemma 1 with the short-selling constraints in* $(A.2)$*. Consider financial innovation that increases the scope of customization for some market participants,* $\tilde{n}^{i_A^1} = n^{i_A^1} + \Delta n$ *and* $\tilde{n}^{i_A^0} = n^{i_A^0} - \Delta n$ *where* $i_A^1 > i_A^0$ *and* $\Delta n > 0$*. This change reduces the risk free rate* r_f*, and leaves unchanged the average risk premia,* $\{\pi_j\}_{j \in \mathbf{J}}$*.*

The result follows by observing that increasing the scope of customization does not affect the characterization of the risk premia in part (i). This is because the maximum excess valuation, $\max_{(\mathbf{i}_A, \mathbf{i}_\mathbf{B})} \Delta_j^{(\mathbf{i}_A, \mathbf{i}_\mathbf{B})}$, remains unchanged before and after the innovation. In contrast, greater customization does affect the characterization of the risk-free rate by enabling more speculation, which reduces the risk-free interest rate as in Proposition 2.

A.4 Customization with Investment

In the main text, we examined the asset pricing implications of financial innovations in an environment with a fixed supply of the market portfolio, m, given by $\eta_m > 0$. In this section, we show that our main general equilibrium result on customization (see Section 5) continues to hold if there is investment and η_m is determined endogenously.

Suppose that the output of the economy at date $t = 1$ is produced via a constant returns to scale neoclassical production function $Y_1 = \Phi(\mathbf{z}) G(K, L)$. Here, we assume that Φ is a Hicks-neutral productivity shock that satisfies,

$$\log \Phi(\mathbf{z}) = \mathbf{F}_m^{'} \mathbf{z}.$$

Investors in this economy are also workers and supply one unit of labor inelastically. Therefore, we modify Assumption $\mathbf{1^G}$ slightly to allow for a positive $t = 1$ endowment by investors. We also maintain the structure of investors' market access and beliefs from Section 5.

The economy starts with zero units of capital at time 0. Capital is produced at time 0 by a competitive sector of investment goods firms that can convert one unit of consumption good at time 0 into one unit of capital at time 1. Since this is only a two-period model, we assume that the capital depreciates fully after use at time 1. Capital and labor are rented at time 1 by a competitive sector of production firms that have access to the production technology of the economy. Given linearity in the investment good technology and production technology for the final good, both types of firms earn zero profits in equilibrium.

We use a similar equilibrium concept for this economy as our "approximate equilibrium" notion in Definition 1 but add additional market clearing conditions that capture the endogeneity of investment. In equilibrium, the price of a unit of capital at $t = 0$ equals its cost of production (namely unity). In addition, the supply of the market portfolio, η_m, equals the supply of capital K. Thus, the price of the market portfolio also equals the price of capital, $P_m = 1$. Finally, there is market clearing in the labor market.

We let $R_m(\eta_m) = \Phi G_K(\eta_m, 1)$ denote the gross return on the market portfolio in equilibrium given supply η_m . We let $r_m(\eta_m)$ denote the log return. Then, the expected log return is given by,

$$E[r_m(\eta_m)] = E[\log \Phi] + \log G_K(\eta_m, 1),$$

Note also that (log) expected return on the market portfolio is equal to the sum of the risk-free rate and the risk premium,

$$E[r_m(\eta_m)] + \frac{\Lambda_m}{2} = r_f + \pi_m.$$

Here, $\Lambda_m = var(\Phi)$ denotes the variance of the market portfolio as in the main text. Combining the last two equations yields the key equation of the characterization,

$$\log G_K(\eta_m, 1) + \left(\frac{\Lambda_m}{2} + E[\log \Phi]\right) = r_f + \pi_m. \tag{A.4}$$

The terms in parentheses are exogenous variables. Hence, the equation says that the supply of capital, η_m, is decreasing in the return on the market portfolio, $r_f + \pi_m$. As we will see, the risk premium on the market portfolio, π_m, will also be determined by exogenous variables. Hence, the equation implies that a lower interest rate, r_f, increases the equilibrium supply of capital, η_m.

To characterize the rest of the equilibrium, we denote the equilibrium wage rate at time 1 by $w(\eta_m) = \Phi G_L(\eta_m, 1)$. Notice that, given the assumption of a Hicks-neutral productivity shock, Φ, the return on the market portfolio and the wage rate are perfectly positively correlated. Since all investors are assumed to have access to the market portfolio, it follows that agents in this economy do not face uninsurable background risks. In particular, the investor's labor endowment is equivalent to holding $\frac{G_L(\eta_m,1)}{G_K(\eta_m,1)}$ units of the market portfolio. Therefore, investor i's effective wealth at time 0 is given by,

$$\tilde{W}_0^i = Y_0 + \frac{G_L(\eta_m, 1)}{G_K(\eta_m, 1)} P_m = Y_0 + \frac{G_L(\eta_m, 1)}{G_K(\eta_m, 1)}.$$

The investor's effective asset holding is given by $\tilde{A}_0^i = a^i\left(r_{ce}^i\right)\tilde{W}_0^i$, where r_{ce}^i is the investor's (log) certainty equivalent return as before. It follows that the investor's savings are given by,

$$A_0^i = a^i\left(r_{ce}^i\right)\tilde{W}_0^i - \frac{G_L(\eta_m, 1)}{G_K(\eta_m, 1)}.$$

The asset market clearing conditions can then be written as,

$$\eta_j P_j = \sum_{\{i | j \in \{f\} \cup J^i\}} n^i \omega_j^i \left[a^i\left(r_{ce}^i\right)\tilde{W}_0^i - \frac{G_L(\eta_m, 1)}{G_K(\eta_m, 1)}\right]. \tag{A.5}$$

The following result characterizes the equilibrium.

Lemma 4. *Consider the setting with limited customization, full participation in the market port-*

folio, belief disagreements that satisfy (14) and (15), and endogenous investment. There exists an equilibrium in which:

(i)The aggregate risk premium on each risky asset satisfies $\pi_j = \frac{\Lambda_{jm}}{\Lambda_m}\pi_m$*, where* $\pi_m = \gamma\Lambda_m$*.*

(ii) The supply of the market portfolio, η_m*, and the risk-free rate,* r_f*, are jointly determined by Eq.* $(A.4)$ *and*

$$\frac{\eta_m + \frac{G_L(\eta_m,1)}{G_K(\eta_m,1)}}{Y_0 + \frac{G_L(\eta_m,1)}{G_K(\eta_m,1)}} = \sum_{i \in I} n^{i_A} n^{\mathbf{i_B}} a\left(r_{ce}^{(i_A,\mathbf{i_B})}\right), \tag{A.6}$$

where the certainty equivalent return for an investor with type $(i_A, \mathbf{i_B})$ *is given by Eq.* (17) *as in the main text.*

Compared to Lemma 1, the only difference is that the supply of the market portfolio, η_m, is endogenous and inversely related to the interest rate according to $(A.4)$. This leads to the following result, which generalizes Proposition 2 to this setting.

Proposition 7. *Consider the equilibrium characterized in Lemma 4. Consider financial innovation that increases the scope of customization for some investors,* $\tilde{n}^{i_A^1} = n^{i_A^1} + \Delta n$ *and* $\tilde{n}^{i_A^0} = n^{i_A^0} - \Delta n$*, where* $i_A^1 > i_A^0$ *and* $\Delta n > 0$*. This change reduces the risk free rate* r_f *and the expected return on risky assets* $E[r_j]$*,* $j \in \mathbf{J}$ *and leaves unchanged the average risk premia. It also increases aggregate investment and the supply of the market portfolio,* η_m*.*

As with the case of a fixed supply of the market portfolio, increased customization decreases the returns on all assets in the economy. In this case, the lower required returns (or the lower hurdle rates) also translate into greater investment and increased supply, η_m. It is illustrative to consider how the two responses compare. The proof in the appendix implies that,

$$\frac{\partial r_f}{\partial \Delta n} / \frac{\partial \eta_m}{\partial \Delta n} = \frac{G_{KK}}{G_K}.$$

Therefore, the relative response depends on properties of the aggregate production function. Specifically, if capital and labor are perfect substitutes in production, then $\frac{G_{KK}}{G_K} = 0$, and only the quantity margin responds. If they are perfect complements, then $\frac{G_{KK}}{G_K} \to \infty$, and there is only a price respond. In between, the relative response should depend on the elasticity of substitution between capital and labor.

A.5 Demand for Safety and Securitization

In this section, we present our formal results on emerging market savings and securitization, which we describe and numerically illustrate in Section 7. Recall that we work with two types of investors, denoted by $\{1, 2\}$, that have common beliefs. Type 2 investors—the EM governments—participate only in the safe asset, $J^2 = \emptyset$, and they have preference parameters, β^2, ε^2, that induce a relatively large asset holdings in equilibrium (as we will formalize below). Type 1 investors optimally choose between direct investment and securitization. The optimality condition for these investors can be written as,

$$\text{if } r_{ce}^{(1,-)} = r_{ce}^{(1,+)} \text{ (resp. } > \text{) (resp. } < \text{), then } n^+ \in (0,1) \text{ (resp. } n^+ = 0\text{) (resp. } n^+ = 1\text{).} \quad \text{(A.7)}$$

The following result characterizes the equilibrium.

Lemma 5. *Consider the above setup. There exists an equilibrium in which* $\pi_m \geq \gamma\Lambda_m, r_f, n^+ \in [0,1]$ *jointly solve,*

$$\pi_m = \gamma\Lambda_m\left(1 + \frac{n^2}{(1-n^2)\,n^+}\frac{a^2(r_f)}{a^1(r_{ce}^1)}\right), \quad \text{(A.8)}$$

$$\frac{\eta_m P_m}{Y_0 + \eta_m P_m} = n^2 a^2(r_f) + \left(1-n^2\right)a^1\left(r_{ce}^1\right), \quad \text{(A.9)}$$

and condition $(A.7)$*, where* $r_{ce}^1 = \max\left(r_{ce}^{(1,-)}, r_{ce}^{(1,+)}\right)$, $r_{ce}^{(1,-)}$ *and* $r_{ce}^{(1,+)}$ *are characterized by Eq.* (24)*.*

The result is similar to Lemma 2 with limited participation, with three differences.[18] The first difference is that the aggregate risk that is not held by type 2 investors is now absorbed by a smaller fraction of investors that choose to securitize. Eq. $(A.8)$ says that the risk premium is determined by the compensation required by the securitizers. A second difference is that type 2 and 1 investors have different asset holding functions, which is captured by the market clearing equation $(A.9)$. The third difference is that the degree of securitization is endogenous and determined by the optimality condition $(A.7)$. Proposition 4 establishes the comparative statics of this equilibrium.

B Appendix B: Omitted Proofs

B.1 Proofs of the results on the consumption and savings problem

This section presents the proofs of the results in Section 3 and Appendices A.1 and A.2.

Proof of Proposition 1. Suppose that the investor has a positive future endowment, $L^i(\mathbf{z})$, and consider a hypothetical investor with zero future endowment, $\tilde{L}^i(\mathbf{z}) = 0$, but instead with financial wealth,

$$\tilde{W}_0^i = \sum_{j \in \{f\} \cup J^i} \left(x_{-1,j}^i + l_j^i\right) P_j.$$

[18] Unlike Lemma 2, we are unable to establish the uniqueness of equilibrium, since $a^2(\cdot)$ and $a^1(\cdot)$ are potentially different, although the equilibrium is unique in all of our numerical simulations.

In view of Assumption 1, given the optimal choice by the hypothetical investor, $\tilde{C}_0^i, \tilde{A}_0^i, \tilde{x}_j^i$, the optimal choice by the original investor can be deduced from,

$$C_0^i = \tilde{C}_0^i, A_0^i = \tilde{A}_0^i - \sum_{j \in \{f\} \cup J^i} l_j^i P_j \text{ and } x_j^i = \tilde{x}_j^i - l_j^i \text{ for each } j.$$

Note also that the borrowing constraint, $A_0^i \geq A^{i,\min}$, implies an analogous constraint for the hypothetical investor, $\tilde{A}_0^i \geq \tilde{A}^{i,\min}$, where $\tilde{A}^{i,\min} = A^{i,\min} + \sum_{j \in \{f\} \cup J^i} l_j^i P_j$.

As discussed in the body of the paper, the (hypothetical) investor's problem can be split into two parts. Conditional on asset holdings, $\tilde{A}_0^i$, the investor maximizes her certainty-equivalent payoff at date 1. That is, she solves the portfolio problem (5). In turn, given the value function $V_1^i(\cdot)$, she chooses her asset holdings, A_0^i, by maximizing the intertemporal utility function in (4). It is straightforward to verify that the portfolio problem in (5) is linearly homogeneous, so $V_1^i(\cdot)$ is a linear function. In particular, $V_1^i(1) = R_{ce}^i$, which gives the investor's certainty equivalent return. The remainder of the proposition follows from the discussion in the main text. □

Proof of Proposition 5. Let $u^i(c) = c^{1-\gamma^i}$ denote the investor's state utility function. First consider the investor's allocation before financial innovation. The optimality condition for the risk-free asset can then be written as,

$$u'\left(C_0^{i,old}\right) = \left(\beta^i / P_f\right) E\left[u'\left(C_1^{i,old}(\mathbf{z})\right)\right].$$

The key observation is that the CRRA preferences satisfy the prudence condition, $u'''(c) > 0$. In view of this observation (and Jensen's inequality), the optimality condition implies,

$$u'\left(C_0^{i,old}\right) \geq \left(\beta^i / P_f\right) u'\left(E\left[C_1^{i,old}(\mathbf{z})\right]\right). \tag{B.1}$$

This expression illustrates the precautionary savings motive. When $\beta^i / P_f = 1$, the investor would like to have greater average consumption in the future compared to the current period. Next consider the investor's allocation after financial innovation. In view of Assumption 1^{P}, and the assumption that financial assets complete the market, the investor chooses the perfect risk sharing allocation, that is, $C_1^{i,new}(\mathbf{z}) \equiv \overline{C}_1^{i,new}$ for each $\mathbf{z} \in Z$. Consequently, the optimality condition for the safe asset implies,

$$u'\left(C_0^{i,new}\right) = \left(\beta^i / P_f\right) u'\left(\overline{C}_1^{i,new}\right). \tag{B.2}$$

Finally, in view of Assumption 1^{P}, the investor's consumption in either case satisfies her lifetime budget constraint,

$$C_0^i + \frac{P_f}{\overline{\varphi}_f} E^n\left[C_1^i(\mathbf{z})\right] = Y_0^i + W_0^i + \frac{P_f}{\overline{\varphi}_f} E^n\left[L_1^i(\mathbf{z})\right].$$

Combining Eqs. $(B.1)$ and $(B.2)$ with the lifetime budget constraint implies that $C_0^{i,new} \geq C_0^{i,old}$, or equivalently, $A_0^{i,new} \leq A_0^{i,old}$. Moreover, the inequality is strict whenever the investor's consumption with old assets features less than perfect risk sharing, completing the proof. □

Proof of Observation 1. To show part (i), we use the linearity in V_1^i and the Euler equation from the intertemporal problem for an investor, for which the borrowing constraint is not binding, to write

$$\tilde{A}_0^i = \max\left\{\frac{\beta^{\varepsilon^i}\left(R_{ce}^i\right)^{\varepsilon^i-1}}{1+\beta^{\varepsilon^i}\left(R_{ce}^i\right)^{\varepsilon^i-1}}\tilde{W}_0^i, \tilde{A}^{i,\min}\right\}. \tag{B.3}$$

Therefore, for an unconstrained investor,

$$\frac{\partial \tilde{A}_0^i}{\partial R_{ce}^i} = \left(\varepsilon^i - 1\right)\left(R_{ce}^i\right)^{\varepsilon^i}\tilde{W}_0^i \frac{\beta^{\varepsilon^i}}{\left[1+\beta^{\varepsilon^i}\left(R_{ce}^i\right)^{\varepsilon^i-1}\right]^2} > 0, \tag{B.4}$$

and similarly, $\frac{\partial \tilde{A}_0^i}{\partial \tilde{W}_0^i} > 0$. Notice that, since $R_{ce}^{i,new} \geq R_{ce}^{i,old}$, an investor that is unconstrained before financial innovation is also unconstrained after financial innovation. However, if an investor is constrained before financial innovation, she may either remain constrained after financial innovation or become unconstrained. Therefore, one has to examine three cases about whether the borrowing constraint binds for an investor before and after financial innovation.

We have $\frac{\partial^2 \tilde{A}_0^i}{\partial R_{ce}^i \partial \tilde{W}_0^i} > 0$. Therefore, since $R_{ce}^{i,new} \geq R_{ce}^{i,old}$, if an investor is initially unconstrained, then,

$$\frac{\partial}{\partial \tilde{W}_0^i}\left(\tilde{A}_0^{i,new} - \tilde{A}_0^{i,old}\right) = \frac{\partial}{\partial \tilde{W}_0^i}\int_{R_{ce}^{i,old}}^{R_{ce}^{i,new}} \frac{\partial \tilde{A}_0^i\left(R, \tilde{W}_0^i\right)}{\partial R_{ce}^i} dR = \int_{R_{ce}^{i,old}}^{R_{ce}^{i,new}} \frac{\partial^2 \tilde{A}_0^i}{\partial R_{ce}^i \partial \tilde{W}_0^i} dR \geq 0.$$

where we can exchange the differentiation and integration since $\frac{\partial \tilde{A}_0^i}{\partial R_{ce}^i}$ is integrable. Since $A_0^{i,new} - A_0^{i,old} = \tilde{A}_0^{i,new} - \tilde{A}_0^{i,old}$, we also have that $A_0^{i,new} - A_0^{i,old}$ is increasing in $\tilde{W}_0^i$, and hence, in W_0^i. Next, if the investor is initially constrained and remains constrained after financial innovation, then

$$A_0^{i,new} - A_0^{i,old} = \tilde{A}_0^{i,new} - \tilde{A}_0^{i,old} = 0,$$

which does not depend on W_0^i. Finally, if the investor becomes unconstrained after financial innovation, then

$$A_0^{i,new} - A_0^{i,old} = A_0^{i,new} - A^{i,\min} = \tilde{A}_0^{i,new} - \tilde{A}^{i,\min},$$

which is increasing in W_0^i by $(B.4)$. It follows that in all three cases $\frac{\partial}{\partial W_0^i}\left(A_0^{i,new} - A_0^{i,old}\right) \geq 0$. Since $\frac{S_0^{i,new} - S_0^{i,old}}{Y_0^i} = \frac{A_0^{i,new} - A_0^{i,old}}{Y_0^i}$, the result follows.

For part (ii), first of all note that

$$R_{ce}^{i,old} = \overline{\varphi}_f P_f^{-1}.$$

does not depend on the value of γ^i. Next, consider two investors, i_1 and i_2 who are identical except for different risk-aversion coefficients given by γ^{i_1} and γ^{i_2}, with $\gamma^{i_1} > \gamma^{i_2}$. Let $\left\{ \tilde{x}_j^{i_1} \left(\tilde{A}_0 \right) \right\}_{\{f\} \cup J^{new}}$ denote the utility maximizing portfolio of investor i_1, given asset holdings $\tilde{A}_0$ and let $C_1^{i_1}(\mathbf{z})$ denote the resulting $t = 1$ consumption for state $\mathbf{z} \in \mathbf{Z}$. Therefore,

$$V_1^{i_1}\left(\tilde{A}_0\right) = R_{ce}^{i_1,new}\tilde{A}_0 = \left(E^{i_1}\left[C_1^{i_1}(\mathbf{z})^{1-\gamma^{i_1}}\right]\right)^{1/\left(1-\gamma^{i_1}\right)}.$$

Notice, however, that $\left(E^{i_1}\left[C_1^{i_1}(\mathbf{z})^{1-\gamma^{i_1}}\right]\right)^{1/\left(1-\gamma^{i_1}\right)}$ is the certainty equivalent consumption for investor i_1, and since investor i_2 is less risk averse in the sense of $\gamma^{i_1} > \gamma^{i_2}$, it follows that

$$R_{ce}^{i_1,new}\tilde{A}_0 = \left(E^{i_1}\left[C_1^{i_1}(\mathbf{z})^{1-\gamma^{i_1}}\right]\right)^{1/\left(1-\gamma^{i_1}\right)} \leq \left(E^{i_1}\left[C_1^{i_1}(\mathbf{z})^{1-\gamma^{i_2}}\right]\right)^{1/\left(1-\gamma^{i_2}\right)} \leq V_1^{i_2}\left(\tilde{A}_0\right) = R_{ce}^{i_2,new}\tilde{A}_0.$$

Thus, $R_{ce}^{i_1,new} \leq R_{ce}^{i_2,new}$. Since $\widetilde{A}_0^i$ and A_0^i are increasing in R_{ce}^i by equation $(B.4)$, and since $R_{ce}^{i_1,old} = R_{ce}^{i_2,old}$ implies $A_0^{i_1,old} = A_0^{i_2,old}$, the result follows. □

B.2 Proofs of results on customization

This section presents the proofs of the results in Section 5 and Appendices A.3 and A.4.

Proof of Lemma 1. We define the average portfolio share of an asset j among all investors that have market access $i_A \in I_A$ as,

$$\omega_j^{i_A} = \frac{\sum_{I_B} n^{\mathbf{i}_B}\omega_j^{(i_A,\mathbf{i}_B)} a\left(r_{ce}^{(i_A,\mathbf{i}_B)}\right)}{\sum_{I_B} n^{\mathbf{i}_B} a\left(r_{ce}^{(i_A,\mathbf{i}_B)}\right)}. \tag{B.5}$$

We will establish the existence of an equilibrium in which prices are uniquely characterized by parts (i)-(ii), investors' certainty-equivalent returns are given by Eq. (17), and their average portfolio shares are given by,

$$\boldsymbol{\omega}_{J^{i_A}}^{i_A} = \frac{1}{\gamma}\Lambda_{J^{i_A}}^{-1}\boldsymbol{\pi}_{J^{i_A}} = [\omega_m, 0, ..0]' \text{ for each } i_A\text{, where } \omega_m = \frac{\pi_m}{\gamma\Lambda_m}. \tag{B.6}$$

Here, $[\omega_m, 0, ..0]$ is a $\left|J^{i_A}\right|$-dimensional vector whose first entry is ω_m and the remaining entries are zero. Hence, in addition to the properties in the lemma, we claim that investors' average portfolio shares are independent of the heterogeneity in beliefs or market access.

We first establish Eq. $(B.6)$, given the prices characterized by parts (i)-(ii) and the certainty-equivalent returns in (17). To prove this, consider an investor's perceived risk premium for a risky asset j, which can be written as,

$$\pi_j^{(i_A,\mathbf{i}_B)} = (\mathbf{F}_j)'\boldsymbol{\mu}_{\mathbf{z}}^i + \frac{\Lambda_j}{2} - \log P_j - r_f = \pi_j + \mathbf{F}_j'\mathbf{i}_B. \tag{B.7}$$

Using Eq. (11), her demand for the risky assets J^{i_A} (as a proportion of her wealth) is given by the vector,

$$\boldsymbol{\omega}_{J^{i_A}}^{(i_A,\mathbf{i}_B)} a\left(r_{ce}^{(i_A,\mathbf{i}_B)}\right) = \frac{1}{\gamma}\Lambda_{J^{i_A}}^{-1}\left(\boldsymbol{\pi}_{J^{i_A}} + \mathbf{F}'_{J^{i_A}}\mathbf{i}_B\right) a\left(r_{ce}^{(i_A,\mathbf{i}_B)}\right).$$

In view of Eq. (17), investors of types $(i_A, \mathbf{i}_B)$ and $(i_A, -\mathbf{i}_B)$ obtain exactly the same certainty equivalent return. Combining these observations, the average demand across belief types $\mathbf{i}_B$ and $-\mathbf{i}_B$ is given by,

$$\frac{\boldsymbol{\omega}_{J^{i_A}}^{(i_A,\mathbf{i}_B)} a\left(r_{ce}^{(i_A,\mathbf{i}_B)}\right) + \boldsymbol{\omega}_{J^{i_A}}^{(i_A,-\mathbf{i}_B)} a\left(r_{ce}^{(i_A,-\mathbf{i}_B)}\right)}{2} = \frac{1}{\gamma}\Lambda_{J^{i_A}}^{-1}\boldsymbol{\pi}_{J^{i_A}} a\left(r_{ce}^{(i_A,\mathbf{i}_B)}\right).$$

Averaging across all belief types, and using Eq. (14), we further obtain,

$$\sum_{\mathbf{i}_B} n^{\mathbf{i}_B}\boldsymbol{\omega}_{J^{i_A}}^{(i_A,\mathbf{i}_B)} a\left(r_{ce}^{(i_A,\mathbf{i}_B)}\right) = \left(\frac{1}{\gamma}\Lambda_{J^{i_A}}^{-1}\boldsymbol{\pi}_{J^{i_A}}\right)\left(\sum_{\mathbf{i}_B} n^{\mathbf{i}_B} a\left(r_{ce}^{(i_A,\mathbf{i}_B)}\right)\right).$$

Using the definition of the average portfolio share in $(B.5)$, we obtain $\boldsymbol{\omega}_{J^{i_A}}^{i_A} = \frac{1}{\gamma}\Lambda_{J^{i_A}}^{-1}\pi_{J^{i_A}}$. Next note that,

$$\left(\Lambda_{J^{i_A}}\left[\omega_m, 0, ..0\right]'\right)_j = \Lambda_{mj}\omega_m = \frac{1}{\gamma}\frac{\Lambda_{mj}\pi_m}{\Lambda_m} = \frac{1}{\gamma}\pi_j,$$

where the last equation uses part (i). Applying $\Lambda_{J^{i_A}}^{-1}$ to both sides of the expression implies, $\boldsymbol{\omega}_{J^{i_A}}^{i_A} = \frac{1}{\gamma}\Lambda_{J^{i_A}}^{-1}\pi_{J^{i_A}} = [\omega_m, 0, ..0]'$, proving Eq. $(B.6)$.

We next check that the investors' certainty-equivalent returns are given by Eq. (17). Using Eqs. (11) and $(B.7)$, we have,

$$\begin{aligned}
r_{ce}^{(i_A,\mathbf{i}_B)} &= r_f + \frac{1}{2\gamma}\left(\boldsymbol{\pi}_{J^{i_A}} + \mathbf{F}'_{J^{i_A}}\mathbf{i}_B\right)'\Lambda_{J^{i_A}}^{-1}\left(\boldsymbol{\pi}_{J^{i_A}} + \mathbf{F}'_{J^i}\mathbf{i}_B\right) \\
&= r_f + \frac{1}{2\gamma}\left(\boldsymbol{\pi}'_{J^{i_A}}\Lambda_{J^{i_A}}^{-1}\boldsymbol{\pi}_{J^{i_A}} + 2\left(\mathbf{F}'_{J^{i_A}}\mathbf{i}_B\right)\left(\Lambda_{J^{i_A}}^{-1}\boldsymbol{\pi}_{J^{i_A}}\right) + \left(\mathbf{F}'_{J^{i_A}}\mathbf{i}_B\right)'\Lambda_{J^{i_A}}^{-1}\left(\mathbf{F}'_{J^{i_A}}\mathbf{i}_B\right)\right) \\
&= r_f + \frac{1}{2}\left(\boldsymbol{\pi}'_{J^{i_A}}\left[\omega_m, 0, ..0\right]'\right) + \frac{1}{2\gamma}\left(\mathbf{F}'_{J^{i_A}}\mathbf{i}_B\right)'\Lambda_{J^{i_A}}^{-1}\left(\mathbf{F}'_{J^{i_A}}\mathbf{i}_B\right) + \left(\mathbf{F}'_{J^{i_A}}\mathbf{i}_B\right)\left[\omega_m, 0, ..0\right]' \\
&= r_f + \frac{1}{2\gamma}\frac{\pi_m^2}{\Lambda_m} + \frac{1}{2\gamma}\left(\mathbf{F}'_{J^{i_A}}\mathbf{i}_B\right)'\Lambda_{J^{i_A}}^{-1}\left(\mathbf{F}'_{J^{i_A}}\mathbf{i}_B\right),
\end{aligned}$$

verifying Eq. (17). Here, the third line uses Eq. $(B.6)$, and the last line uses the assumption (15) that there is no disagreement on the market portfolio, so that $(\mathbf{F}_m)'\,\mathbf{i}_B = 0$.

Next note that parts (i)-(ii) uniquely characterize the equilibrium prices of all assets. We finally check that these prices satisfy the $|\mathbf{J}|+1$ market clearing conditions (13). The conditions for $j \neq m$ hold because $\omega_j^{i_A} = 0$ for each i_A and $j \neq m$. To check the remaining conditions, substitute $\omega_m = 1$ in view of part (i). After this substitution, the market clearing condition for asset f holds since each investor has a zero weight on the risk-free asset, $\omega_f = 1 - \omega_m = 0$. The market clearing condition for asset m also holds, since the condition becomes identical to Eq. (16) in part (ii). This establishes the existence of an equilibrium that satisfies the conditions in the lemma along with Eq. $(B.6)$, completing the proof. □

Proof of Proposition 2. To show this result, notice that a direct extension of the revealed preference argument underlying the Choice Channel (Proposition **1**) to the case of a continuous state space implies that $R_{ce}^{i_A^1} \geq R_{ce}^{i_A^0}$, and hence, $r_{ce}^{(i_A^1, \mathbf{i}_B)} \geq r_{ce}^{(i_A^0, \mathbf{i}_B)}$. Alternatively, a direct inspection of Eq. (17) and the observation that $\left(\mathbf{F'}_{J^{i_A}}(\mathbf{i}_B)\right)' \Lambda_{J^{i_A}}^{-1} \left(\mathbf{F'}_{J^{i_A}}(\mathbf{i}_B)\right)$, the square of the speculative Sharpe ratio (Simsek, 2013b), is higher for the i_A^1 investor also gives the same result. Next, re-write (16) as

$$\sum_{i \in I} n^{i_A} n^{\mathbf{i}_B} a\left(r_{ce}^{(i_A, \mathbf{i}_B)}\right) - \frac{\eta_m P_m}{Y_0 + \eta_m P_m} = 0 \tag{B.8}$$

and notice that the left-hand side is increasing in r_f, since $r_{ce}^{(i_A, \mathbf{i}_B)}$ is increasing in r_f and $a(.)$ is an increasing function, so the first term is increasing in r_f, and also P_m is decreasing in r_f, so the second term is also increasing in r_f. Finally, since $r_{ce}^{(i_A^1, \mathbf{i}_B)} \geq r_{ce}^{(i_A^0, \mathbf{i}_B)}$, it follows that $\sum_{i \in I} n^{i_A} n^{\mathbf{i}_B} a\left(r_{ce}^{(i_A, \mathbf{i}_B)}\right)$ is increasing in Δn, and so, the left-hand side of $(B.8)$ is increasing in Δn. Hence, r_f is decreasing in Δn.

Showing that $\{\pi_j\}_{j \in \mathbf{J}}$ remain unchanged follows directly from Lemma **1** (i). Finally, showing that the average expected return on risky assets decreases follows from the behavior of r_f and $\{\pi_j\}_{j \in \mathbf{J}}$. □

Proof of Lemma 3. First, we show that given prices characterized by parts (i)-(ii) and the certainty equivalent return (20), average portfolio shares for investors with market access i_A, are independent of the heterogeneity in beliefs or market access and satisfy Eq. $(B.6)$. An investor's perceived risk premium for a risky asset j is

$$\pi_j^{(i_A, \mathbf{i_B})} = \mathbf{F}_j' \boldsymbol{\mu}_z^i + ((\Lambda_j)/2) - \log P_j - r_f = \pi_j + \mathbf{F}_j' \mathbf{i_B}. \tag{B.9}$$

The first-order conditions for the investor can be written as

$$\boldsymbol{\pi}_{J^{i_A}}^{(i_A, \mathbf{i_B})} - \gamma \Lambda_{J^{i_A}} \boldsymbol{\omega}_{J^{i_A}}^{(i_A, \mathbf{i_B})} + \boldsymbol{\kappa}_{J^{i_A}}^{(i_A, \mathbf{i_B})} = 0, \tag{B.10}$$

where $\boldsymbol{\kappa}_{J^{i_A}}^{(i_A, \mathbf{i_B})}$ consists of a $|\tilde{J}^{i_A}|$-by-1 vector $\boldsymbol{\kappa}_{\tilde{J}^{i_A}}^{(i_A, \mathbf{i_B})}$ of Lagrange multipliers for the respective short-selling constraints and a $|J^{i_A} \backslash \tilde{\mathbf{J}}|$-by-1 vector of zeros for the assets that do not have short-selling constraints. Substituting for $\pi_j^{(i_A, \mathbf{i_B})}$ from (B.9), we have,

$$\boldsymbol{\pi}_{J^{i_A}} + \mathbf{F}_{J^{i_A}}' \mathbf{i_B} - \gamma \Lambda_{J^{i_A}} \boldsymbol{\omega}_{J^{i_A}}^{(i_A, \mathbf{i_B})} + \boldsymbol{\kappa}_{J^{i_A}}^{(i_A, \mathbf{i_B})} = 0. \tag{B.11}$$

Next, we show that $\boldsymbol{\omega}_{J^{i_A}}^{(i_A,\mathbf{i_B})} = \left[\mathbf{0}_{|\tilde{J}^{i_A}|}, \left(\boldsymbol{\omega}_{J^{i_A}\backslash\tilde{\mathbf{J}}}^{(i_A,\mathbf{i_B})}\right)'\right]'$ satisfies the FOCs in (B.11), where

$$\boldsymbol{\omega}_{J^{i_A}/\tilde{\mathbf{J}}}^{(i_A,\mathbf{i_B})} = \left(\frac{1}{\gamma}\Lambda_{J^{i_A}\backslash\tilde{\mathbf{J}}}^{-1}\left(\boldsymbol{\pi}_{J^{i_A}\backslash\tilde{\mathbf{J}}} + \left(\mathbf{F}_{J^{i_A}\backslash\tilde{\mathbf{J}}}\right)'\mathbf{i_B}\right)\right), \tag{B.12}$$

To show this, first note that for assets in $J^{i_A}\backslash\tilde{\mathbf{J}}$ we have

$$\boldsymbol{\pi}_{J^{i_A}\backslash\tilde{\mathbf{J}}} + \left(\mathbf{F}_{J^{i_A}\backslash\tilde{\mathbf{J}}}\right)'\mathbf{i_B} - \gamma\Lambda_{J^{i_A}\backslash\tilde{\mathbf{J}}}\boldsymbol{\omega}_{J^{i_A}\backslash\tilde{\mathbf{J}}}^{(i_A,\mathbf{i_B})} = 0,$$

which is satisfied given the definition of $\boldsymbol{\omega}_{J^{i_A}\backslash\tilde{\mathbf{J}}}^{(i_A,\mathbf{i_B})}$. Also, notice that the individual weights in $\boldsymbol{\omega}_{J^{i_A}\backslash\tilde{\mathbf{J}}}^{(i_A,\mathbf{i_B})}$ are equivalent to the optimal individual portfolio weights in an equilibrium in which only $\mathbf{J}\backslash\tilde{\mathbf{J}}$ assets are available, with risk premia given in (i), and the individual investor i_A has access to $J^{i_A}\backslash\tilde{\mathbf{J}}$ of those. Therefore, the results from Lemma 1 apply for the average portfolio weights, $\boldsymbol{\omega}_{J^{i_A}/\tilde{\mathbf{J}}}^{i_A}$, across investor with different beliefs, and so, a version of Eq. $(B.6)$ holds for these average weights. This in turn implies that we can simplify (B.12) to

$$\boldsymbol{\omega}_{J^{i_A}/\tilde{\mathbf{J}}}^{(i_A,\mathbf{i_B})} = \boldsymbol{\omega}_{J^{i_A}/\tilde{\mathbf{J}}}^{i_A} + \frac{1}{\gamma}\Lambda_{J^{i_A}\backslash\tilde{\mathbf{J}}}^{-1}\left(\mathbf{F}_{J^{i_A}\backslash\tilde{\mathbf{J}}}\right)'\mathbf{i_B}. \tag{B.13}$$

For assets $j \in \tilde{J}^{i_A}$, these weights imply that

$$\begin{aligned}
\boldsymbol{\kappa}_j^{(i_A,\mathbf{i_B})} &= -\pi_j - \left[\mathbf{F}_j'\mathbf{i_B} - \left(\gamma\tilde{\Lambda}_{\tilde{J}^{i_A}}\boldsymbol{\omega}_{J^{i_A}\backslash\tilde{\mathbf{J}}}^{(i_A,\mathbf{i_B})}\right)_j\right] \\
&= -\pi_j - \left[\mathbf{F}_j'\mathbf{i_B} - \left(\gamma\tilde{\Lambda}_{\tilde{J}^{i_A}}\boldsymbol{\omega}_{J^{i_A}/\tilde{\mathbf{J}}}^{i_A} + \tilde{\Lambda}_{\tilde{J}^{i_A}}\Lambda_{J^{i_A}\backslash\tilde{\mathbf{J}}}^{-1}\left(\mathbf{F}_{J^{i_A}\backslash\tilde{\mathbf{J}}}\right)'\mathbf{i_B}\right)_j\right] \\
&= -\pi_j + \frac{\Lambda_{jm}}{\Lambda_m}\pi_m - \left[\mathbf{F}_j'\mathbf{i_B} - \tilde{\Lambda}_{j,\tilde{J}^{i_A}}\Lambda_{J^{i_A}\backslash\tilde{\mathbf{J}}}^{-1}\left(\mathbf{F}_{J^{i_A}\backslash\tilde{\mathbf{J}}}\right)'\mathbf{i_B}\right] \geq 0,
\end{aligned}$$

where the last inequality follows given the equilibrium values of π_j in (i).

To show the rest of the lemma, we use the observation that the individual portfolio weights on assets without short-selling constraints are equivalent to those in an equilibrium in which only $\mathbf{J}\backslash\tilde{\mathbf{J}}$ assets are available, with risk premia given in (i), and the individual investor i_A has access to $J^{i_A}\backslash\tilde{\mathbf{J}}$ of those. Therefore, an application of Lemma 1 implies that the investors' certainty-equivalent returns are given by equation (20). Finally, Lemma 1 implies that parts (i) and (ii) uniquely characterize the equilibrium prices on all assets and also all market clearing conditions are satisfied at these prices. □

Proof of Proposition 6. The result on assets $\mathbf{J}\backslash\tilde{\mathbf{J}}$ follows by observing that investors' portfolio weights and certainty-equivalent returns are equivalent to those in an equilibrium in which only $\mathbf{J}\backslash\tilde{\mathbf{J}}$ assets are available, and an individual investor with market access i_A has access to $J^{i_A}\backslash\tilde{\mathbf{J}}$ of those, and applying Proposition 2 to the environment with $\mathbf{J}\backslash\tilde{\mathbf{J}}$ available assets**.** The result on the remaining assets $\tilde{\mathbf{J}}$ follow by applying Lemma 3 before and after customization, and observing that $\max_{(i_A,\mathbf{i_B})} \Delta_j^{(\mathbf{i}_A,\mathbf{i_B})}$ is the same in both cases. □

Proof of Lemma 4. We proceed along the lines of the proof of Lemma 1. Specifically, we show that there exists an equilibrium in which the risk premia, risk-free rate, and the supply of the market portfolio are uniquely determined by (i) and (ii). First, observe that the endogenous supply of the market portfolio does not affect any investor's portfolio problem directly, but only indirectly through the equilibrium prices. Therefore, given equilibrium prices, average portfolio shares for investors with market access $i_A \in I_A$, $\boldsymbol{\omega}_{J^{i_A}}^{i_A}$, defined in (B.5) still satisfy (B.6) and investors' certainty equivalent returns are given by (17).

Next, note that conditions (i) and (ii) still uniquely characterize the equilibrium returns of all assets and the supply of the market portfolio. To see this, notice that after substituting $\pi_m = \gamma\Lambda_m$, Eq. $(A.4)$ describes a downward sloping relation between r_f and η_m in (r_f, η_m)-space. In addition, condition (A.6) describes an upward sloping relation, since the left-hand side of (A.6) is increasing in η_m given that $Y_0 \geq \eta_m$ in equilibrium, and the right-hand side of (A.6) is increasing in r_f since $a(.)$ is assumed to be an increasing function. Also, there exists a solution to (i) and (ii) since by (A.6) and an Inada condition for the production function, $\lim_{\eta_m \to 0} r_f(\eta_m) = -\infty$ and also $\lim_{\eta_m \to Y_0} r_f(\eta_m) = \infty$.

Finally, we check that these returns and the supply of the market portfolio satisfy the market clearing conditions (A.5). The market clearing conditions for $j \neq m$ are clearly satisfied since $\omega_j^{i_A} = 0$, for each i_A and $j \neq m$. The market clearing condition for the risk-free asset is also satisfied since $\omega_m = 1$ and $\omega_f = 1 - \omega_m$, so each investor has a zero weight on the risk-free asset. Finally, the market clearing condition for asset m is equivalent to (A.6) in part (ii), so it also holds. □

Proof of Proposition 7. First, as in Proposition 2, we have $r_{ce}^{(i_A^1,\mathbf{i_B})} \geq r_{ce}^{(i_A^0,\mathbf{i_B})}$. Next, we re-write (A.6) as

$$\sum_{i \in I} n^{i_A} n^{\mathbf{i_B}} a\left(r_{ce}^{(i_A,\mathbf{i_B})}\right) - \frac{\eta_m + \frac{G_L(\eta_m,1)}{G_K(\eta_m,1)}}{Y_0 + \frac{G_L(\eta_m,1)}{G_K(\eta_m,1)}} = 0.$$

Implicitly differentiating this equation (while keeping r_f constant), we obtain,

$$\frac{\partial \eta_m}{\partial \Delta n} \propto \frac{\sum_{\mathbf{i_B}} n^{\mathbf{i_B}} \left[a\left(r_{ce}^{(i_A^1,\mathbf{i_B})}\right) - a\left(r_{ce}^{(i_A^0,\mathbf{i_B})}\right)\right]}{Y_0 + \frac{G_L(\eta_m,1)}{G_K(\eta_m,1)} + \frac{\partial}{\partial \eta_m}\left(\frac{G_L(\eta_m,1)}{G_K(\eta_m,1)}\right)(Y_0 - \eta_m)} \geq 0. \tag{B.14}$$

Here, we use the fact that $\frac{\partial}{\partial \eta_m}\left(\frac{G_L(\eta_m,1)}{G_K(\eta_m,1)}\right) \geq 0$ and $\eta_m \leq Y_0$ in equilibrium. The proof of Lemma 4 shows that r_f and η_m are jointly determined by Eqs. $(A.4)$ and (A.6), which are respectively downward and upward sloping in (r_f, η_m)-space. Eq. (B.14) implies that an increase in Δn leads to an upward shift in relation (A.6). This in turn implies that the equilibrium value of r_f declines and the value of η_m increases. By part (i) of Lemma 4, the decline in r_f translates into a decline in the expected return on all assets. $\square$

B.3 Proofs of results on participation

This section presents the proofs of the results in Section 6. We start with useful lemma about the behavior of the asset holding function.

Lemma 6. *Whenever $\varepsilon > 1$, the semi-elasticity $\frac{a'(r_{ce})}{a(r_{ce})}$ is decreasing in r_{ce}.*

Proof. From the Euler Equation in logarithmic form

$$\log a(r_{ce}) - \log(1 - a(r_{ce})) = \varepsilon \log \beta + (\varepsilon - 1) r_{ce}$$

thus differentiating with respect to r_{ce} and simplifying

$$\frac{a'(r_{ce})}{a(r_{ce})} = (\varepsilon - 1)(1 - a(r_{ce})) \tag{B.15}$$

so $\frac{a'(r_{ce})}{a(r_{ce})}$ is decreasing in a and therefore in r_{ce}, whenever $\varepsilon > 1$. $\square$

Proof of Lemma 2. To simplify notation, we leave implicit the dependence of $\omega_1(\pi_m)$,$r^1_{ce}(r_f, \pi_m)$ and $P_m(r_f + \pi_m)$ on (r_f, π_m). Using Eq. (11), type 1 investors' portfolio share and return are given by,

$$\omega^1_m = \frac{\pi_m}{\gamma \Lambda_m} \text{ and } r^1_{ce} = r_f + \frac{1}{2\gamma\Lambda_m}\pi^2_m,$$

establishing Eq. (23).

Notice that the market clearing condition for the safe asset can be written as,

$$0 = n^1\left(1 - \omega^1_m\right) a\left(r^1_{ce}\right) + n^0 a(r_f).$$

Rearranging this expression implies Eq. (21). Finally, Eq. (22) follows by adding all of the market clearing conditions (13).

It remains to show that the system in $(21)-(22)$ has a unique solution. Towards that end let us first define the average level of savings out of wealth as $\overline{a}\left(r_f, \pi_m, n^1\right) \equiv n^1 a\left(r^1_{ce}\right) + \left(1 - n^1\right) a(r_f)$, and the relative value of the asset endowment as $v(r_f + \pi_m) \equiv \frac{\eta_m P_m}{Y_0 + \eta_m P_m}$. Combined they characterize

$$\varphi_1(r_f, \pi_m, n_1) \equiv \overline{a}\left(r_f, \pi_m, n^1\right) - v(r_f + \pi_m).$$

Notice that $v'(r_f + \pi_m) \propto -Y_0 v(r_f + \pi_m) < 0$. As a consequence, $\frac{\partial \varphi_1(r_f, \pi_m, n_1)}{\partial r_f} = \frac{\partial \overline{a}}{\partial r_f} - v' > 0$, and $\frac{\partial \varphi_1(r_f, \pi_m, n_1)}{\partial \pi_m} = \frac{\partial \overline{a}}{\partial \pi_m} - v' > 0$.

Additionally, we define

$$\varphi_2(r_f,\pi_m,n_1) \equiv n^1\left(1-\omega^1\right)a\left(r^1_{ce}\right)+\left(1-n^1\right)a\left(r_f\right).$$

An equilibrium then is a solution to $\varphi_1(r_f,\pi_m,n_1)=\varphi_2(r_f,\pi_m,n_1)=0$.

Notice then that, $\frac{\partial\varphi_2}{\partial r_f}=n^1\left(1-\omega^1\right)a'\left(r^1_{ce}\right)+\left(1-n^1\right)a'\left(r_f\right)$. Additionally, $\varphi_2(r_f,\pi_m,n_1)=0 \implies \left(1-\omega^1\right)=-\frac{\left(1-n^1\right)}{n^1}\frac{a(r_f)}{a(r^1_{ce})}$ and $\frac{\partial\varphi_2}{\partial r_f}=\left(1-n^1\right)a\left(r_f\right)\left[\frac{a'(r_f)}{a(r_f)}-\frac{a'(r_{ce})}{a(r_{ce})}\right]$ which is positive whenever $\varepsilon>1$, given Lemma 6. Last, $\frac{\partial\varphi_2}{\partial\pi_m}=-\frac{\partial\omega^1}{\partial\pi_m}n^1a\left(r^1_{ce}\right)+n^1\left(1-\omega^1\right)a'\left(r^1_{ce}\right)\frac{\partial r_{ce}}{\partial\pi_m}<0$ since $\left(1-\omega^1\right)<0$ whenever $\varphi_2=0$.

As a consequence, locus $\varphi_1(r_f,\pi_m,n_1)=0$ is downward slopping in (r_f,π_m)-space while locus $\varphi_2(r_f,\pi_m,n_1)=0$ is upward slopping. Both loci are characterized by continuous functions. We can use $\varphi_1(r_f,\pi_m,n_1)=0$, with $\frac{\partial\varphi_1}{\partial\pi_m}\neq 0$, and the Implicit Function Theorem to define a decreasing function $\pi^{\varphi_1}_m(\cdot)$ of the interest rate r_f over the first locus. We then look for a solution to $\varphi_2\left(r_f,\pi^{\varphi_1}_m(r_f),n_1\right)=0$, where the left-hand side is a strictly increasing function of r_f. The existence of a solution is guaranteed by Proposition **8** and uniqueness follows from strict monotonicity. □

Proof of Proposition 3 Let $J\equiv\begin{bmatrix}\frac{\partial\varphi_1}{\partial r_f} & \frac{\partial\varphi_1}{\partial\pi_m}\\ \frac{\partial\varphi_2}{\partial r_f} & \frac{\partial\varphi_2}{\partial\pi_m}\end{bmatrix}$ and $\Delta_J<0$ denote its determinant. Then,

$$\begin{bmatrix}\frac{dr_f}{dn^1}\\ \frac{d\pi_m}{dn^1}\end{bmatrix}=-\frac{1}{\Delta_J}\begin{bmatrix}\frac{\partial\varphi_2}{\partial\pi_m} & -\frac{\partial\varphi_1}{\partial\pi_m}\\ -\frac{\partial\varphi_2}{\partial r_f} & \frac{\partial\varphi_1}{\partial r_f}\end{bmatrix}\begin{bmatrix}a\left(r^1_{ce}\right)-a\left(r_f\right)\\ -\frac{a(r_f)}{n_1}\end{bmatrix}.$$

Therefore, $\frac{d\pi_m}{dn^1}<0$. Also,

$$\begin{aligned}\frac{d\left[r_f+\pi_m\right]}{dn^1} &\propto \left(a\left(r^1_{ce}\right)-a\left(r_f\right)\right)\left(\frac{\partial\varphi_2}{\partial\pi_m}-\frac{\partial\varphi_2}{\partial r_f}\right)+\left(\frac{\partial\varphi_1}{\partial\pi_m}-\frac{\partial\varphi_1}{\partial r_f}\right)\frac{a\left(r_f\right)}{n_1}\\ &= \left(a\left(r^1_{ce}\right)-a\left(r_f\right)\right)\left(\frac{\partial\varphi_2}{\partial\pi_m}-\frac{\partial\varphi_2}{\partial r_f}\right)+\left(\frac{a'\left(r^1_{ce}\right)}{a\left(r^1_{ce}\right)}-\frac{a'\left(r_f\right)}{a\left(r_f\right)}\right)\frac{\left(1-n^1\right)}{n_1}\left(a\left(r_f\right)\right)^2<0\end{aligned}$$

again using Lemma 6. □

B.4 Proofs of results on securitization

This section presents the proofs of the results in Section 7 and Appendix A.5.

Proof of Lemma 5. Agents that invest directly face a short-selling constraint for the riskless asset, having therefore a portfolio share given by $\omega^{(1,-)}_m=\max\left(\frac{\pi_m}{\gamma\Lambda_m},1\right)$. In the conjectured equilibrium (and in fact, in any equilibrium), we have $\pi_m>\gamma\Lambda_m$. Thus, the constraint binds and we have,

$$\omega_m^{(1,-)} = 1 \text{ and } r_{ce}^{(1,-)}\left(r_f, \pi_m\right) = r_f + \pi_m - \frac{\gamma}{2}\Lambda_m.$$

Agents that opt for intermediated investment, pay a competitive fee of c per-unit of investment (in terms of log-returns), but are not subject to this constraint. Therefore, the optimal portfolio share in the risky asset and return net of fees are given by

$$\omega_m^{(1,+)}\left(\pi_m\right) = \frac{\pi_m}{\gamma\Lambda_m} > 1 \text{ and } r_{ce}^{(1,+)}\left(r_f, \pi_m\right) = r_f + \frac{1}{2\gamma\Lambda_m}\pi_m^2 - c.$$

Individuals of type 1 endogenously choose whether to become subtype $(1,-)$ or $(1,+)$, therefore solving $r_{ce} = \max\left\{r_{ce}^{(1,-)},\ r_{ce}^{(1,+)}\right\}$ and leading to the equilibrium condition

$$n^+ = \begin{cases} 1, & if\ \pi_m > \overline{\pi}_m, \\ \in [0,1]\,, & if\ \pi_m = \overline{\pi}_m, \\ 0, & if\ \pi_m < \overline{\pi}_m, \end{cases} \tag{B.16}$$

where $\overline{\pi}_m$ is the largest solution to $\frac{1}{2\gamma\Lambda_m}\overline{\pi}_m^2 - \overline{\pi}_m + \frac{\gamma}{2}\Lambda_m = c, i.e., \overline{\pi}_m = \gamma\Lambda_m\left(1 + \sqrt{\frac{2c}{\gamma\Lambda_m}}\right)$.[19]

Next, we define the average levels of savings as $\overline{a}\left(r_f, \pi_m, n^2\right) = n^2 a^2\left(r_f\right) + \left(1 - n^2\right) a^1\left(r_{ce}^1\right)$ and the relative value of asset endowments as $v\left(r_f + \pi_m\right) \equiv \frac{\eta_m P_m}{Y_0 + \eta_m P_m}$. Therefore, $v^{'}\left(r_f + \pi_m\right) \propto -Y_0 v\left(r_f + \pi_m\right)$. We have now a system characterized by two equations derived from the market-clearing conditions

$$\varphi_1\left(r_f, \pi_m, n^2\right) \equiv \overline{a}\left(r_f, \pi_m\right) - v\left(r_f + \pi_m\right) = 0, \tag{B.17}$$

$$\varphi_2\left(r_f, \pi_m, n^+, n^2\right) \equiv n^2 a^2\left(r_f\right) + \left(1 - n^2\right) n^+ \left(1 - \omega^{(1,+)}\right) a^1\left(r_{ce}\right) = 0, \tag{B.18}$$

and optimality in endogenous securitization (B.16). For the characterization, it is convenient to work with three cases.

In all three cases, it is useful to work with a pair of implicit functions. Implicit function $r_f^{\varphi_1}\left(\pi_m, n^2\right)$ is defined over $\varphi_1\left(r_f, \pi_m, n^2\right) = 0$. This function is continuous, monotonically decreasing in π_m, and satisfies $\lim_{\pi_m \to \infty} r_f^{\varphi_1}\left(\pi_m, n^2\right) = -\infty$ and $\lim_{\pi_m \to -\infty} r_f^{\varphi_1}\left(\pi_m, n^2\right) = \infty$. For $n^2 < 1$, monotonicity is strict and an inverse exists. We call this other implicit function $\pi_m^{\varphi_1}\left(r_f, n^2\right)$.

The first case is $n^+ = 0$ and $\pi_m < \overline{\pi}_m$, leading to $n^2 a^2\left(r_f\right) = 0$, which can only hold with $n^2 = 0$. In this case, $\pi_m = \gamma\Lambda_m$ and r_f is the solution to $a^1\left(r_f + \frac{\gamma\Lambda_m}{2}\right) = v\left(r_f + \gamma\Lambda_m\right)$.

Second, we analyze cases where $n^2 > 0$. For these, $r_{ce}\left(r_f, \pi_m\right) = r_{ce}^{(1,+)}\left(r_f, \pi_m\right)$ and it is useful

[19] The smallest root makes $\omega \geq 1$, as opposed to $\omega \leq 1$ bind.

to define $\pi_m^{\varphi_2}\left(r_f,n^+,n^2\right)$ for $n^2,n^+>0$, a well-defined function implicit in $\varphi_2\left(r_f,\pi_m,n^+,n^2\right)=0$, which although not monotonic in r_f, is continuous, is decreasing in n^+, and satisfies $\lim_{r_f\to\infty}\pi_m^{\varphi_2}\left(r_f,n^+,n^2\right)\geq\gamma\Lambda_m>-\infty$ and $\lim_{r_f\to-\infty}\pi_m^{\varphi_2}\left(r_f,n^+,n^2\right)<\infty$.

A first possibility is $\pi_m=\overline{\pi}_m$ and $n^+\in[0,1]$, leading to $r_f^*=r_f^{\varphi_1}\left(\overline{\pi}_m,n^2\right)$. Also, (B.18) allows us to write

$$n^{+,*}\left(\overline{\pi}_m,n^2\right)=\frac{n^2a^2\left(r_f^{\varphi_1}\left(\overline{\pi}_m,n^2\right)\right)}{\sqrt{\frac{2c}{\gamma\Lambda_m}}\left(1-n^2\right)a^1\left(r_{ce}\left(r_f^{\varphi_1}\left(\overline{\pi}_m,n^2\right),\overline{\pi}_m\right)\right)}. \tag{B.19}$$

Whenever $n^{+,*}\left(\overline{\pi}_m,n^2\right)<1$ we have characterized an equilibrium. That equilibrium is the unique interior equilibrium. With $n^{+,*}\left(\overline{\pi}_m,n^2\right)=1$ existence is still ensured. Whenever $n^{+,*}\left(\overline{\pi}_m,n^2\right)>1$ we need to establish existence of an equilibrium in which $n^+=1$ and $\pi_m\geq\overline{\pi}_m$.

Towards that end, notice that for any $n^+\in R_+$, $\lim_{r_f\to\infty}\pi_m^{\varphi_1}\left(r_f,n^2\right)<\lim_{r_f\to\infty}\pi_m^{\varphi_2}\left(r_f,n^+,n^2\right)$ and $\lim_{r_f\to-\infty}\pi_m^{\varphi_1}\left(r_f,n^2\right)>\lim_{r_f\to+\infty}\pi_m^{\varphi_2}\left(r_f,n^+,n^2\right)$. Due to continuity of both functions, they cross at least once. Let $\pi_m^{cross}\left(n^+,n^2\right)$ denote the premium at the crossing with the lowest r_f. There, $\pi_m^{\varphi_2}$ crosses $\pi_m^{\varphi_1}$ from below, and since $\frac{\partial\pi_m^{\varphi_2}}{\partial n^+}<0$, $\pi_m^{cross}\left(n^+,n^2\right)$ is strictly decreasing in n^+. From $n^{+,*}\left(\overline{\pi}_m,n^2\right)>1$ and the definition of $n^{+,*}\left(\overline{\pi}_m,n^2\right)$, we obtain $\pi_m^{cross}\left(1,n^2\right)>\pi_m^{cross}\left(n^{+,*}\left(\overline{\pi}_m,n^2\right),n^2\right)=\overline{\pi}_m$. So, whenever $n^{+,*}\left(\overline{\pi}_m,n^2\right)>1$, the first crossing of $\pi_m^{\varphi_1}\left(r_f,n^2\right)$ and $\pi_m^{\varphi_2}\left(r_f,1,n^2\right)$ establishes the existence of a corner equilibrium where $n^+=1$. □

Proof of Proposition 4. If a solution features $n^+<1$, then by continuity, locally any solution to (B.16)-(B.17)-(B.18) features $\pi_m=\overline{\pi}_m$. That leads to $\varphi_1\left(r_f,\overline{\pi}_m,n^2\right)=\varphi_2\left(r_f,\overline{\pi}_m,n^+,n^2\right)=0$, as in the second case described in the previous proof. From $r_f^*=r_f^{\varphi_1}\left(\overline{\pi}_m,n^2\right)$ defined in the previous proof, $\frac{dr_f^*}{dn^2}<0$ whenever $a^2\left(r_f\right)\geq a^1\left(r_f+\overline{\pi}_m\right)$ is satisfied. In this case, from (B.19),

$$\frac{dn^+}{dn^2}=\frac{\partial n^+}{\partial n^2}+\frac{\partial n^+}{\partial r_f}\frac{dr_f}{dn^2}$$

In this expression, $\frac{\partial n^+}{\partial n^2}=\frac{n^+}{n^2(1-n^2)}>0$, $\frac{\partial n^+}{\partial r_f}=n^+\left[\frac{a^{2\prime}}{a^2}-\frac{a^{1\prime}}{a^1}\right]\leq0$ whenever $a^2\left(r_f\right)\geq a^1\left(r_f+\overline{\pi}_m\right)$ and $\epsilon^2\leq\epsilon^1$, and $\frac{\partial r_f}{\partial n^2}=-\frac{a_2-a_1}{a^{\prime}-v^{\prime}}<0$.

For the second part we have that, $\frac{d\pi_m}{dc}=\frac{d\overline{\pi}_m}{dc}>0$, thus $\frac{dr_f}{dc}=-\frac{\frac{\partial\varphi_1}{\partial\pi_m}}{\frac{\partial\varphi_1}{\partial r_f}}\frac{\partial\pi_m}{\partial c}<0$ and

$$\frac{dn^+}{dc}=\frac{\partial n^+}{\partial c}+\frac{\partial n^+}{\partial r_f}\frac{dr_f}{dc}+\frac{\partial n^+}{\partial\overline{\pi}_m}\frac{\partial\overline{\pi}_m}{\partial c}$$

where $\frac{\partial n^+}{\partial c}<0$, $\frac{\partial n^+}{\partial \overline{\pi}_m}=-n^+\left(\frac{a^{1\prime}}{a^1}\right)<0$ and $\frac{\partial n^+}{\partial r_f}=n^+\left[\frac{a^{2\prime}}{a^2}-\frac{a^{1\prime}}{a^1}\right]$ which leads us to

$$\frac{dn^+}{dc}=\frac{\partial n^+}{\partial c}-n^+\left(\frac{a^{2\prime}}{a^2}\frac{\frac{\partial \varphi_1}{\partial \pi_m}}{\frac{\partial \varphi_1}{\partial r_f}}+\frac{a^{1\prime}}{a^1}\left(1-\frac{\frac{\partial \varphi_1}{\partial \pi_m}}{\frac{\partial \varphi_1}{\partial r_f}}\right)\right)\frac{\partial \overline{\pi}_m}{\partial c}<0,$$

as $\left(1-\frac{\frac{\partial \varphi_1}{\partial \pi_m}}{\frac{\partial \varphi_1}{\partial r_f}}\right)>0$. □

C Appendix C: Omitted Results

Proposition 8 (Existence)**.** *Under Assumptions 1^G and 2, there exists an approximate equilibrium with $P_j>0$ for each $j\in\{f\}\cup\mathbf{J}$. The portfolio weights and the certainty equivalent returns are characterized by Eq.* (11)*, and the prices are characterized as the solution to the demand system* (13)*.*

Proof. Let $P=\{P_j\}_{j\in\{f\}\cup J}$ denote the asset price vector. We work with a truncated economy, where prices satisfy $P_j\leq\alpha$ for each asset $j\in\{f\}\cup J$. We are only interested in sufficiently large α so that the truncation becomes inconsequential. First, let extended portfolio weights be also defined over assets that agent i cannot trade, so that

$$\hat{\omega}_j^i(P)\equiv\begin{cases}\omega_j^i(P), & \text{whenever } j\in\{f\}\cup J^i\\ 0, & \text{otherwise.}\end{cases}$$

For $P\gg0$ we have individual excess demand for asset $j\in\{f\}\cup J$ defined as

$$z_j^i(P)\equiv\frac{\hat{\omega}_j^i(P)}{P_j}A_0^i(P)-x_{-1,j}^i \tag{C.1}$$

and we analogously define the excess demand for consumption at date $t=0$ as $z_0^i(P)\equiv c_0^i(P)-Y_0^i$. Aggregate excess demands are then simply defined as $z_j(P)\equiv\sum_i n^i z_j^i(P)$ and $z_0(P)\equiv\sum_i n^i z_0^i(P)$. Walras' Law, i.e., $z_0(P)+\sum_{j\in J}P_j z_j(P)=0$ can be trivially verified from individual optimality.

First, we impose a lower bound on prices $\hat{\epsilon}>0$, which we successively relax later. Define $S_{\hat{\epsilon}}\equiv\left\{P\in\mathbb{R}_{++}^{|J|}|P_j\geq\hat{\epsilon}\text{ and }P_j\leq\alpha,\forall j\in\{f\}\cup J\right\}$ which is compact and convex. We are only interested in $\alpha>\hat{\epsilon}$ as to ensure the non-emptiness of $S_{\hat{\epsilon}}$.

We next define a continuous price updating function. Let each entry, which describes the update to the price of asset $j\in J$, be defined by

$$P_j^{upd}(P,\hat{\epsilon})\equiv\begin{cases}\hat{\epsilon},\ if\ z_j(P)<\hat{\epsilon}-P_j\\ P_j+z_j(P),\ if\ \hat{\epsilon}-P_j\leq z_j(P)\leq\alpha\\ \alpha,\ if\ z_j(P)>\alpha\end{cases}. \tag{C.2}$$

Then, let the function $P^{upd}(P,\hat{\epsilon}): S_{\hat{\epsilon}} \to S_{\hat{\epsilon}}$ be defined as $P^{upd}(P,\hat{\epsilon}) = \left\{P_j^{upd}(P,\hat{\epsilon})\right\}_{j\in\{f\}\cup J}$. As excess demand functions are continuous, so is the function $P^{upd}(\cdot,\hat{\epsilon})$, which maps the non-empty, convex, and compact set $S_{\hat{\epsilon}}$ into itself. From Brouwer's Fixed Point Theorem, there exists $P^{\hat{\epsilon}} \in S_{\hat{\epsilon}}$ such that $P_j^{upd}\left(P^{\hat{\epsilon}},\hat{\epsilon}\right) = P^{\hat{\epsilon}}$.

We now take a sequence $\{\hat{\epsilon}_k\}_{k\in\mathbb{N}}$ such that $\hat{\epsilon}_k \to 0$. Let $\left\{P^{\hat{\epsilon}_k}\right\}_{k\in\mathbb{N}}$ be the associated sequence of fixed points. As each price lies in $[0,\alpha]$ that sequence is bounded and admits a converging subsequence. To save on notation, assume we have selected such subsequence from the start. Define its limit by $P^* = \left(P_1^*, P_2^*, ..., P_{|J|}^*\right)$. Naturally $P^* \in \overline{\cup_k S_{\hat{\epsilon}_k}} = \left\{P \in \mathbb{R}_+^{|J|} | P_j \leq \alpha, \forall \{f\} \cup J\right\}$. We now show that $P^* \in \mathbb{R}_{++}^{|J|}$.

Consider the case with $P_j^* = 0$ for risky assets, which w.l.o.g. we call assets $1,..,m$, while the riskless rate remains bounded away from zero. In this case, the risk premia for assets $1,..,m$ approach $+\infty$, and the risk premia for the remaining assets remain finite. Consider all investors that have access to at least one of the assets $1,..,m$ and call that set $I_{r\to\infty}$. It is easy to check that each of these investors have $r_{ce} \to \infty$, and thus, they save all their wealth.

Now consider the net demand for assets that comes from these investors only, $z_j^{I_{r\to\infty}} \equiv \sum_{i\in I_{r\to\infty}} n^i z_j^i(P)$. We claim that regardless of how the prices for $1,..,m$ approach 0 (or conversely, regardless of the risk premia approach infinity), there exists at least one asset within $1,..,m$ such that the total demand from these investors for that asset becomes unboundedly positive. Since the demand from the other investors is finite, this will provide a contradiction.

Let us rewrite risk premia along the sequence. Take a given agent $i \in I_{r\to\infty}$, then the (individually perceived) risk-premium $\pi_j^{i,k}\left(P^{\hat{\epsilon}_k}\right)$ on any asset $j \in J$ can be appropriately rewritten as $\pi_j^{i,k} = \left\|\pi^{i,k}\right\| \hat{\pi}_j^{i,k}$ where $\left\|\pi^{i,k}\right\| := \sum_j \left|\pi_j^{i,k}\right|$ denotes a norm and

$$\hat{\pi}_j^{i,k} \equiv \frac{\pi_j^{i,k}}{\|\pi^{i,k}\|}$$

denotes the $j-th$ entry of a normalized risk-premium vector.[20] The vector $\hat{\pi}^{i,k} = \left\{\hat{\pi}_j^{i,k}\right\}_{j\in J}$ belongs to the surface of the unit ball centered at zero.

As that surface is a compact set, $\left\{\hat{\pi}^{i,k}\right\}_{k\in\mathbb{N}}$ admits a converging subsequence, which we can index by $k_i \in \mathbb{N}$. That forms another price sequence $\left\{P^{\hat{\epsilon}_{k_i}}\right\}_{k_i\in\mathbb{N}}$, from which we can extract a subsequence to ensure that the analogously defined vector $\hat{\pi}^{i',k_i}$ converges for any second agent $i' \in I_{r\to\infty}$. Given that $I_{r\to\infty}$ is finite, this step can be iteratively repeated until a subsequence, indexed by $\tilde{k} \in \mathbb{N}$, is extracted and ensures that each $\hat{\pi}^{i,\tilde{k}}$ converges. Additionally, for each $i \in I_{r\to\infty}$, $\lim_{\tilde{k}\to\infty} \hat{\pi}^{i,\tilde{k}} = \hat{\pi}$, i.e., the limit of the normalized risk-premia are the same and independent of $i \in I_{r\to\infty}$, since disagreements are bounded, while at least one return goes to infinity.

Take a given agent $i \in I_{r\to\infty}$. Define $\hat{\pi}_{J_i}^{i,\tilde{k}}$ and $\hat{\pi}_{J_i}$ to be respectively the restriction of the

[20] As prices are converging to zero, there are finitely many elements with $\sum_j \pi_j^{i,k} = 0$. We can move to a subsequence that disregards these.

normalized risk premia vectors $\hat{\pi}^{i,\tilde{k}}$ and $\hat{\pi}$ to the assets that agent i can trade. Notice that along that subsequence portfolio weights of the form $\omega^i_{J^i}\left(P^{\hat{\epsilon}_{\tilde{k}}}\right) = \frac{1}{\gamma^i}\Lambda^{-1}_{J^i}\hat{\pi}^{i,\tilde{k}}_{J_i}\left\|\pi^{i,\tilde{k}}\right\|$ are optimal from equation (11). Therefore, we take the following limit of an inner product

$$\lim_{\tilde{k}\to\infty}\left\langle \hat{\pi}^{i,\tilde{k}}_{J_i}, \frac{\omega^i_{J^i}\left(P^{\hat{\epsilon}_{\tilde{k}}}\right)}{\left\|\pi^{i,\tilde{k}}\right\|}\right\rangle = \frac{1}{\gamma^i}\hat{\pi}'_{J_i}\Lambda^{-1}_{J^i}\hat{\pi}_{J_i} > 0$$

from the positive-definiteness of $\Lambda^{-1}_{J^i}$ and the fact that $\hat{\pi}_{J_i}$ is not null. It follows that it is possible to find $\delta > 0$ and a sufficiently large element $\overline{k}$ such that

$$\left\langle \hat{\pi}, \frac{\hat{\omega}^i\left(P^{\hat{\epsilon}_{\tilde{k}}}\right)}{\left\|\pi^{i,\tilde{k}}\right\|}\right\rangle > \delta,$$

whenever $i \in I_{r\to\infty}$ and $\tilde{k} > \overline{k}$. Given that $A^i_0\left(P^{\hat{\epsilon}_{\tilde{k}}}\right)$ is bounded from below for sufficiently high $\tilde{k}$ for all $i \in I_{r\to\infty}$, there exists $\delta_1 > 0$

$$\left\langle \hat{\pi}, \sum_{i\in I_{r\to\infty}} n^i A^i_0\left(P^{\hat{\epsilon}_{\tilde{k}}}\right)\frac{\hat{\omega}^i\left(P^{\hat{\epsilon}_{\tilde{k}}}\right)}{\left\|\pi^{i,\tilde{k}}\right\|}\right\rangle > \delta_1, \tag{C.3}$$

for all $\tilde{k} > \overline{k}$. This directly implies that there exists one asset $j \in \{1,...,m\}$ such that $\sum_{i\in I_{r\to\infty}} n^i A^i_0\left(P^{\hat{\epsilon}_{\tilde{k}}}\right)\hat{\omega}^i\left(P^{\hat{\epsilon}_{\tilde{k}}}\right)$ grows without bounds. It follows that excess demand for that asset is unbounded along the subsequence that is indexed by $\tilde{k}$. From (C.2) this means that $P^{upd}_j\left(P^{\hat{\epsilon}_k}, \hat{\epsilon}_k\right) = \alpha$ infinitely many times as $k \to \infty$, reaching a contradiction with $P^*_j = 0$.

Suppose now, towards a different contradiction, that $r_f \to \infty$. Using arguments similar to the previous ones, it is possible to select a subsequence, indexed by $\tilde{k} \in \mathbb{N}$, in which the risk premium, $\pi^i_j\left(P^{\hat{\epsilon}_{\tilde{k}}}\right)$, perceived by each agent $i \in I$ for each asset $j \in J$ either converges to a finite constant, diverges to $+\infty$ or diverges to $-\infty$. Also, a premium can only diverge for all agents at the same time and in the same direction.

First, we deal with the case in which no premium diverges. In this situation, each asset price converges to zero. Adding equations C.1 over agents and assets, properly multiplied by prices and individual population shares, we get

$$\sum_{i,j} P^{\hat{\epsilon}_{\tilde{k}}}_j n^i z^i_j\left(P^{\hat{\epsilon}_{\tilde{k}}}\right) = \sum_{i,j} n^i\left[\hat{\omega^i_j}\left(P^{\hat{\epsilon}_{\tilde{k}}}\right)A^i_0\left(P^{\hat{\epsilon}_{\tilde{k}}}\right) - P^{\hat{\epsilon}_{\tilde{k}}}_j x^i_{-1,j}\right],$$

which after simplifications leads to

$$\sum_j P^{\hat{\epsilon}_{\tilde{k}}}_j z_j\left(P^{\hat{\epsilon}_{\tilde{k}}}\right) = \sum_i n^i A^i_0\left(P^{\hat{\epsilon}_{\tilde{k}}}\right) - \sum_{i,j} P^{\hat{\epsilon}_{\tilde{k}}}_j n^i x^i_{-1,j}.$$

As $P^{\hat{\epsilon}_{\tilde{k}}} \to 0$, the right hand side converges to $\sum_i n^i Y^i_0 > 0$. As a consequence, the excess demand for at least one asset j needs to approach $+\infty$ along a subsequence. Along this subsequence then $P_j^{upd}\left(P, \hat{\epsilon}_{\tilde{k}}\right) = \alpha$ infinitely often, leading to a contradiction of the zero price limit.

For the case in which some premia diverge, we still obtain

$$\lim_{\tilde{k}\to\infty} A_0^i\left(P^{\hat{\epsilon}_{\tilde{k}}}\right) - P_j^{\hat{\epsilon}_{\tilde{k}}} n^i x_{-1,j}^i = Y_0^i > 0$$

and

$$\sum_j P_j^{\hat{\epsilon}_{\tilde{k}}} z_j\left(P^{\hat{\epsilon}_{\tilde{k}}}\right) \to \sum n^i Y_0^i > 0$$

If $P^{\hat{\epsilon}_{\tilde{k}}} \to 0$, we find the same contradiction as before. Therefore, for at least one asset $j \in J$, we need to have $P_j^{\hat{\epsilon}_{\tilde{k}}} \to P_j^* \neq 0$ which implies that $\pi_j^{i,\tilde{k}} \to -\infty$ for each $i \in I$. We can therefore follow all the previous steps leading to C.3, with the exception that $\hat{\pi}$ can now have negative entries. This means that we can find a subsequence and an asset $j' \in J$, such that either $\pi_{j'}^{i,\tilde{k}} \to -\infty$ and $z_{j'}\left(P^{\hat{\epsilon}_{\tilde{k}}}\right) \to -\infty$ or $\pi_{j'}^{i,\tilde{k}} \to +\infty$ and $z_{j'}\left(P^{\hat{\epsilon}_{\tilde{k}}}\right) \to +\infty$. For the latter case, we would reach the same contradiction as before since $z_{j'}\left(P^{\hat{\epsilon}_{\tilde{k}}}\right) \to +\infty$ implies that $P_{j'}^{upd}\left(P^{\hat{\epsilon}_k}, \hat{\epsilon}_k\right) = \alpha$ infinitely many times which contradicts positive infinity limits for both the riskless rate and the risk premium on j'. Therefore, we need to rule out the former situation. Given that $P_j^{\hat{\epsilon}_{\tilde{k}}} \to P_j^* > 0$, $\pi_j^{i,\tilde{k}} \to -\infty$ and $\hat{\pi}_{j'} \neq 0$ together imply that $P_{j'}^* > 0$. But from (C.2), $z_{j'}\left(P^{\hat{\epsilon}_{\tilde{k}}}\right) \to -\infty$ implies $P_{j'}^{\hat{\epsilon}_{\tilde{k}}} = \hat{\epsilon}_{\tilde{k}}$ infinitely many times with $\hat{\epsilon}_{\tilde{k}} \to 0$, reaching a contradiction with $P_{j'}^* > 0$.

We have, therefore, ruled out any possibility that $P_j^* = 0$ for some asset $j \in J \cup \{f\}$. We still need to show that for sufficiently high α, market clearing is ensured in all markets at prices P^*. Given that $P_j^* \gg 0$, it is possible to find a sufficiently high $\hat{k}$ and $\delta_2 > 0$, such that

$$P_j^{\hat{\epsilon}_k} > \delta_2 > \hat{\epsilon}_k,$$

for all $k > \hat{k}$. As a consequence, from (C.2), for $k > \hat{k}$, $P_j^{\hat{\epsilon}_k} \geq 0$ and $z_j\left(P^{\hat{\epsilon}_k}\right) \geq 0$.

Additionally, for each $i \in I$, $C_0^i\left(P^{\hat{\epsilon}_k}\right) \in \left[0, Y_0^i + \alpha \sum_j x_{-1}^i, j\right]$ implying that

$$-\alpha \sum_{i,j} n^i x_{-1,j}^i \leq \sum_j P_j^{\hat{\epsilon}_{\tilde{k}}} z_j\left(P^{\hat{\epsilon}_{\tilde{k}}}\right) \leq \sum_i n^i Y_0^i.$$

For $\alpha^2 > \sum_i n^i Y_0^i$, it follows that $z\left(P^{\hat{\epsilon}_{\tilde{k}}}\right) \to z\left(P^*\right) = 0$ ensuring market-clearing in the limit and existence of a Walrasian Equilibrium.

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