Uniqueness of immersed spheres in three-manifolds, and a conjecture by Alexandrov

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Resumo/Abstract:
A famous theorem by Hopf proves that any constant mean curvature sphere in R3 is a round sphere. In this talk we will generalize Hopf's theorem to classes of surfaces modeled by arbitrary elliptic PDEs in arbitrary three-manifolds, with the only hypothesis of the existence of a family of "candidate surfaces" within the class. In this way, we prove that any immersed sphere in such a class of surfaces is a candidate sphere. As an application, we prove a 1956 conjecture by A.D. Alexandrov on the uniqueness of immersed spheres of prescribed curvatures in R3, and we complete the characterization of round spheres as the only elliptic Weingarten spheres in R3 (Weingarten spheres are immersed spheres in R3 whose principal curvatures are linked by a non-trivial elliptic relation). Joint work with P. Mira