

On compact almost Ricci soliton with Cotton tensor identically zero

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Abstract:

The concept of almost Ricci soliton (ARS) was introduced by Pigola et al. in [7], where essentially they modified the definition of a Ricci soliton by requiring that the parameter λ to be a variable function, i.e., $Ric + \frac{1}{2}L_X g = \lambda g$, where Ric and L stand for the Ricci curvature tensor and the Lie derivative, respectively.

We point out that a Ricci soliton also corresponds to self-similar solutions of Hamilton's Ricci flow, see e.g. [4]. In the same spirit Brozos-Vázquez, García-Río and Valle-Regueiro [3] observed that some proper gradient almost Ricci solitons correspond to self-similar solutions of the Ricci-Bourguignon flow, which is given by

$$\frac{\partial}{\partial t} g(t) = -2(Ric(t) - kR(t)g(t)),$$

where k is a real number and R stands for the scalar curvature. This flow can be seen as an interpolation between the flows of Ricci and Yamabe. For more details we recommend [5].

It is important to be emphasized that the round sphere does not admit a (nontrivial) Ricci soliton structure. However, Barros and Ribeiro Jr [1] proved that there exists a unique gradient ARS structure on a standard sphere with constant scalar curvature. Afterward, Barros, Batista and Ribeiro Jr [2] proved that every compact ARS with constant scalar curvature is gradient.

Our aim here is to use the result obtained in [1] to present the following result.

Teorema Let (M^n, g, X, λ) be a non-trivial compact oriented ARS with Cotton tensor identically zero. Then, M^n is isometric to a standard sphere provided that one of the next condition is satisfied:

- (1) $S_2(A)$ is constant and positive.

(2) $S_k(A)$ is nowhere zero on M and $S_{k+1}(A) = cS_k(A)$, where c is a non null constant, for some $k = 1, \dots, n - 1$.

(3) $S_k(A)$ is constant for some $k = 2, \dots, n - 1$, and $A > 0$, where A stands for the Schouten and $\sigma_k(A)$ be the symmetric functions associated to A .

We point out that symmetric functions associated to the Schouten tensor were used by Li and Simon [6] to study locally conformally flat manifolds. In addition, the first and the third conditions above improve Theorem 1 in [6] for compact ARS under the hypothesis of Cotton tensor identically zero.

References

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