Some optimal decay estimates for damped semi-linear evolution equations

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Resumo/Abstract:

In this talk we discuss the critical exponent for global small data solutions to the $\sigma-$ evolution equation with structural effective damping

$$u_{tt} + 2\mu(-\Delta)^{\delta}u_t + (-\Delta)^{\sigma}u = ||D|^a\partial_t^k u|^p, \quad t > 0, x \in R^n,$$
$$(u, u_t)(0, x) = (u_0, u_1)(x),$$

with $\mu > 0$, $2\delta \in (0, \sigma)$, $k = 0, 1$ and $a \in [0, \sigma + 2\delta)$. According to different cases, we fix data regularity and solution spaces which allow us to derive optimal decay estimates for the solution, in the sense that the obtained decay rate is the same of the linear case. In particular, we show which benefits come by assuming smallness of the initial data in spaces on $L^1 \cap L^\infty$ basis, when the fractional power of the damping term is close to the limit case at which the damping cease to be effective. We prepare several high frequencies estimates for the corresponding linear problem, applying Mikhlin-Hörmander type multiplier theorems and Riesz potential properties. We couple these estimates with low frequencies estimates, which optimality is guaranteed by the diffusion phenomenon, and we deal with the nonlinear term by using a standard contraction argument in a suitable solution space.

References

[1] M. D'Abbicco, M. R. Ebert, *Some optimal decay estimates for damped semi-linear evolution equations*, Submitted, 2016.

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