

FOUR-MANIFOLDS WITH PINCHED POSITIVE SECTIONAL CURVATURE

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Resumo/Abstract:

We prove that a four-dimensional compact oriented connected Riemannian manifold whose sectional curvatures all lie in the interval $[\frac{3\sqrt{3}-1}{26}, 1]$ is necessarily definite. In particular, we improve the pinching constants considered by some preceding works. In addition, we show that a four-dimensional compact oriented Einstein manifold whose sectional curvatures all lie in the interval $[\frac{1}{10}, 1]$ is either topologically S^4 or homothetically isometric to CP^2 , equipped with its standard Fubini-Study metric.

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