About the Hardy-Sobolev, Moser-Trudinger and isoperimetric inequalities with densities

Gyula Csató [1,3] Prosenjit Roy[2]

[1]Universidad de Concepción, Chile
[2]Department of Mathematics, Technion, Haifa, Israel

Abstract:
The standard isoperimetric inequality states that among all sets with a given fixed volume the ball has the smallest perimeter. That is, written here for simplicity in dimension 2, the following infimum is attained by the ball

$$2\pi r = \inf\left\{\int_{\partial\Omega} 1\, d\sigma(x) : \Omega \subset \mathbf{R}^2 \quad \text{and} \quad \int_{\Omega} 1\, dx = \pi r^2\right\}.$$

The isoperimetric problem with density is a generalization of this question: given two positive functions $f, g : \mathbf{R}^2 \to \mathbf{R}$ one studies the existence of minimizers of

$$I(C) = \inf\left\{\int_{\partial\Omega} g(x) d\sigma(x) : \Omega \subset \mathbf{R}^2 \quad \text{and} \quad \int_{\Omega} f(x) dx = C\right\}.$$

I will talk about some results when $f(x) = |x|^q$ and $g(x) = |x|^p$, where $p, q \in \mathbf{R}$. This is a rich problem with strong variations in difficulties depending on the values of p and q. I will first give an overview on Sobolev, Hardy-Sobolev and Moser-Trudinger inequalities and establish different kind of connections to isoperimetric inequalities with densities. Finally I will present some of the results appearing in the following references:

References

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[2] Csató G. and Roy P., *Extremal functions for the singular Moser-Trudinger inequality in 2 dimensions*, Calc. Var. Partial Differential Equations, **54**, Issue 2 (2015), 2341–2366.

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