

Minimal surfaces in 4-dimensional Lorentzian Damek-Ricci spaces

Adriana A. Cintra¹, Francesco Mercuri², Irene I. Onnis³

¹ UFG

² Unicamp

³ USP

Resumo/Abstract:

Damek-Ricci spaces are semidirect products of Heisenberg groups with the real line. They were considered in [2] (see also [1]), equipped with a left-invariant Riemannian metric, to give a negative answer, in high dimensions, to the question posed by Lichnerowicz: “is a harmonic Riemannian manifold necessarily a symmetric space?” Beside a left-invariant Riemannian metric, these spaces may be equipped with left-invariant Lorentzian metrics in essentially two ways: a Riemannian metric on the Heisenberg factor and a negative metric on the \mathbb{R} factor, or a Lorentzian metric in the Heisenberg factor and a positive metric on \mathbb{R} . In [5] the authors give a Weierstrass representation theorem for minimal surfaces in Riemannian manifolds. In [4] this representation has been extended for timelike and spacelike minimal surfaces in 3-dimensional Lorentzian manifolds. The results can be easily extended to the case of minimal surfaces in Lorentzian manifolds of higher dimension. Most of the applications and examples of these results are given for 3-dimensional ambient spaces. For higher dimension, there is an application of this formula for minimal surfaces in 4-dimensional Damek-Ricci spaces equipped with a left-invariant Riemannian metric (see [3]). The aim is to study a Weierstrass representation for simply connected minimal surfaces in these spaces, in dimension four.

References

- [1] J. Berndt, F. Tricerri, L. Vanhecke. *Generalized Heisenberg Groups and Damek-Ricci Harmonic Spaces*, Lecture Notes in Mathematics, **1598**, Springer-Verlag, Berlin, (1991).

- [2] E. Damek, F. Ricci. *A class of nonsymmetric harmonic Riemannian spaces*, Bull. Amer. Math. Soc. **27** (1992), 139-142.
- [3] M. Koivogui, L. Todjihounde. *Weierstrass Representation for minimal immersions into Damek-Ricci spaces*, Int. Electron. J. Geom. **6** (2013), 1-7.
- [4] J.H. Lira, M. Melo, F. Mercuri. *A Weierstrass representation for minimal surfaces in 3-dimensional manifolds*, Results. Math. **60** (2011), 311-323.
- [5] F. Mercuri, S. Montaldo, P. Piu. *A Weierstrass representation formula of minimal surfaces in \mathbb{H}_3 and $\mathbb{H}^2 \times \mathbb{R}$* , Acta Math. Sinica **22** (2006), 1603-1612.
- [6] A. Cintra, F. Mercuri, I. Onnis, *Minimal surfaces in 4-dimensional Lorentzian Damek-Ricci spaces*, <http://arxiv.org/abs/1501.03427>.