

Index and topology of minimal hypersurfaces in \mathbb{R}^n

Chao Li ,

Stanford University

Abstract:

In this paper, we consider immersed two-sided minimal hypersurfaces in \mathbb{R}^n with finite total curvature. We prove that the sum of the Morse index and the nullity of the Jacobi operator is bounded from below by a linear function of the number of ends and the first Betti number of the hypersurface. When $n = 4$, we are able to drop the nullity term by a careful study for the rigidity case. Our result is the first effective generalization of [13]. Using our index estimates and ideas from the recent work of Chodosh-Ketover-Maximo [3], we prove compactness and finiteness results of minimal hypersurfaces in \mathbb{R}^4 with finite index.

REFERENCES

- [1] Serge Bernstein. Über ein geometrisches theorem und seine anwendung auf die partiellen differentialgleichungen vom elliptischen typus. *Mathematische Zeitschrift*, 26(1):551–558, 1927.
- [2] Huai-Dong Cao, Ying Shen, and Shunhui Zhu. The structure of stable minimal hypersurfaces in \mathbb{R}^{n+1} . *Mathematical Research Letters*, 4(5):637–644, 1997.
- [3] Otis Chodosh, Daniel Ketover, and Davi Maximo. Minimal surfaces with bounded index. *arXiv preprint arXiv:1509.06724*, 2015.
- [4] Jaigyoung Choe. Index, vision number and stability of complete minimal surfaces. *Archive for Rational Mechanics and Analysis*, 109(3):195–212, 1990.
- [5] Celso J. Costa. Example of a complete minimal immersion in \mathbb{R}^3 of genus one and three-embedded ends. *Boletim da Sociedade Brasileira de Matemática - Bulletin/Brazilian Mathematical Society*, 15(1):47–54, 1982.
- [6] Manfredo do Carmo and Chia-Kuei Peng. Stable complete minimal surfaces in \mathbb{R}^3 are planes. *Bulletin of the American Mathematical Society*, 1(6):903–906, 1979.
- [7] Doris Fischer-Colbrie. On complete minimal surfaces with finite morse index in three manifolds. *Inventiones Mathematicae*, 82(1):121–132, 1985.
- [8] Doris Fischer-Colbrie and Richard Schoen. The structure of complete stable minimal surfaces in 3-manifolds of non-negative scalar curvature. *Communications on Pure and Applied Mathematics*, 33(2):199–211, 1980.
- [9] David Hoffman and William H Meeks. Embedded minimal surfaces of finite topology. *Annals of Mathematics*, 131(1):1–34, 1990.
- [10] David A. Hoffman and William Meeks, III. A complete embedded minimal surface in \mathbb{R}^3 with genus one and three ends. *J. Differential Geom.*, 21(1):109–127, 1985.
- [11] William C. Jagy. Minimal hypersurfaces foliated by spheres. *Michigan Math. J.*, 38(2):255–270, 1991.
- [12] Luquésio P Jorge and William H Meeks. The topology of complete minimal surfaces of finite total gaussian curvature. *Topology*, 22(2):203–221, 1983.
- [13] Peter Li and Jiaping Wang. Minimal hypersurfaces with finite index. *Mathematical Research Letters*, 9(1):95–104, 2002.
- [14] Davi Maximo and Otis Chodosh. On the topology and index of minimal surfaces. *arXiv:1405.7356*, 2014.
- [15] Jiaqiang Mei and Senlin Xu. On minimal hypersurfaces with finite harmonic indices. *Duke Math. J.*, 110(2):195–215, 11 2001.
- [16] J. H. Michael and L. M. Simon. Sobolev and mean-value inequalities on generalized submanifolds of \mathbb{R}^n . *Communications on Pure and Applied Mathematics*, 26(3):361–379, 1973.

- [17] Sebastián Montiel and Antonio Ros. Schrödinger operators associated to a holomorphic map. In *Global differential geometry and global analysis*, pages 147–174. Springer, 1991.
- [18] Filippo MORABITO. Index and nullity of the gauss map of the costa-hoffman-meeks surfaces. *Indiana University mathematics journal*, 58(2):677–707, 2009.
- [19] Shin Nayatani. Morse index of complete minimal surfaces. *The problem of Plateau, World Sci. Publ., River Edge, NJ*, pages 181–189, 1992.
- [20] A.V. Pogorelov. On the stability of minimal surfaces. *Sov. Math., Dokl.*, 24:274–276, 1981.
- [21] Antonio Ros. One-sided complete stable minimal surfaces. *Journal of Differential Geometry*, 74(1):69–92, 2006.
- [22] Alessandro Savo. Index bounds for minimal hypersurfaces of the sphere. *Indiana University mathematics journal*, 59(3):823–837, 2010.
- [23] Richard Schoen and Leon Simon. Regularity of stable minimal hypersurfaces. *Communications on Pure and Applied Mathematics*, 34(6):741–797, 1981.
- [24] Richard Schoen and Shing Tung Yau. Harmonic maps and the topology of stable hypersurfaces and manifolds with non-negative ricci curvature. *Commentarii Mathematici Helvetici*, 51(1):333–341, 1976.
- [25] Richard M. Schoen. Uniqueness, symmetry, and embeddedness of minimal surfaces. *J. Differential Geom.*, 18(4):791–809, 1983.
- [26] Ben Sharp. Compactness of minimal hypersurfaces with bounded index. *arXiv preprint arXiv:1501.02703*, 2015.
- [27] Yi-Bing Shen and Xiao-Hua Zhu. On stable complete minimal hypersurfaces in. *American Journal of Mathematics*, pages 103–116, 1998.
- [28] Luen-Fai Tam and Detang Zhou. Stability properties for the higher dimensional catenoid in \mathbb{R}^{n+1} . *Proceedings of the American Mathematical Society*, 137(10):3451–3461, 2009.
- [29] Shukichi Tanno. l^2 harmonic forms and stability of minimal hypersurfaces. *Journal of the Mathematical Society of Japan*, 48(4):761–768, 1996.
- [30] Johan Tysk. Finiteness of index and total scalar curvature for minimal hypersurfaces. *Proceedings of the American Mathematical Society*, 105(2):429–435, 1989.
- [31] Brian White. Curvature estimates and compactness theorems in 3-manifolds for surfaces that are stationary for parametric elliptic functionals. *Inventiones mathematicae*, 88(2):243–256, 1987.