THIRD-ORDER DIFFERENTIAL EQUATIONS AND LOCAL ISOMETRIC IMMERSIONS OF PSEUDOSPHERICAL SURFACES

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Abstract. : The class of differential equations describing pseudospherical surfaces enjoys important integrability properties which manifest themselves by the existence of infinite hierarchies of conservation laws (both local and non-local) and the presence associated linear problems. It thus contains many important known examples of integrable equations, like the sine-Gordon, Liouville, KdV, mKdV, Camassa-Holm and Degasperis-Procesi equations, and is also home to many new families of integrable equations. Our paper is concerned with the question of the local isometric immersion in $\mathbf{E}^3$ of the pseudospherical surfaces defined by the solutions of equations belonging to the class introduced by Chern and Tenenblat [3]. In the case of the sine-Gordon equation, it is a classical result that the second fundamental form of the immersion depends only on a jet of finite order of the solution of the pde. A natural question is therefore to know if this remarkable property extends to equations other than the sine-Gordon equation within the class of differential equations describing pseudospherical surfaces. In a pair of earlier papers [13], [14] it was shown that this property fails to hold for all k-th order evolution equations $u_t = F(u, u_x, ..., u_{x^k})$ and all other second order equations of the form $u_{xt} = F(u, u_x)$, except for the sine-Gordon equation and a special class of equations for which the coefficients of the second fundamental form are universal, that is functions of x and t which are independent of the choice of solution u. In the present paper, we consider third order equations of the form $u_t - u_{xxt} = \lambda u u_{xxx} + G(u, u_x, u_{xx})$, $\lambda \in \mathbb{R}$, which describe pseudospherical surfaces with the Riemannian metric given in [2]. This class contains the Camassa-Holm and Degasperis-Procesi equations as special cases. We show that whenever there exists a local isometric immersion in $\mathbf{E}^3$ for which the coefficients of the second fundamental form depend on a jet of finite order of u, then these coefficients are universal in the sense of being independent on the choice of solution u. This result further underscores the special place that the sine-Gordon equations seems to occupy amongst integrable partial differential equations in one space variable.

Keywords: Nonlinear partial differential equations; pseudospherical surfaces; local isometric immersion

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Referências

[1] Beals, R.; Rabelo, M.; Tenenblat, K. *Backlünd transformations and inverse scattering for some pseudo-spherical surface equations*. Stud. Appl. Math. **81** (1989), 125-151.

[2] Castro Silva, T.; Tenenblat, K. *Third order differential equations describing pseudospherical surfaces.* J. Diff. Eq. **259** (2015), 4897-4923.

[3] Chern, S. S.; Tenenblat, K. *Pseudospherical surfaces and evolution equations.* Stud. Appl. Math. **74** (1986), 55-83.

[4] Camassa, R.; Holm, D. D. *An integrable shallow water equation with peaked solitons.* Phys. Rev. Lett. **71** (11) (1993), 1661-1664.

[5] Cavalcante, D., and Tenenblat, K. *Conservation laws for nonlinear evolution equations.* J. Math. Phys. **29** (1988), 1044-1049.

[6] Catalano Ferraioli, D.; Tenenblat, K. *Fourth order evolution equations which describe pseudospherical surfaces.* J. Diff. Eq. **257** (2014), 3165-3199.

[7] Degasperis, A; Procesi, M. *Asymptotic Integrability, in Symmetry and Perturbation Theory.* Editors: Degasperis A and Gaeta G, World Scientific, Singapore (1999), 23-37.

[8] Foursov, V., Olver, P.J., and Reyes, E. *On formal integrability of evolution equations and local geometry of surfaces.* Diff. Geom. Appl. **15** (2001), 183-199.

[9] Gomes Neto, V. P. *Fifth-order evolution equations describing pseudospherical surfaces.* J. of Differential Equations **249** (2010), 2822-2865.

[10] Gorka, P., and Reyes, E. *The modified Hunter-Saxton equation.* J. Geom. Phys. **62** (2012), 1793-1809.

[11] Jorge, L., and Tenenblat, K. *Linear problems associated to evolution equations of type $u_{tt} = F(u, u_x, \ldots, u_{x^k})$.* Stud. Appl. Math. **77** (1987), 103-117,

[12] Kamran, N.; Tenenblat, K. *On differential equations describing pseudo-spherical surfaces.* J. Differential Equations **115** (1995), 75-98.

[13] Kahouadji, N.; Kamran, N.; Tenenblat, K. *Second-order equations and local isometric immersions of pseudo-spherical surfaces.* arXiv: 1308.6545, 25 pages. To appear in Comm. Analysis and Geometry, 2015.

[14] Kahouadji, N.; Kamran, N.; Tenenblat, K. *Local isometric immersions of pseudo-spherical surfaces and evolution equations.* Hamiltonian Partial Differential Equations and Applications, Guyenne, P.; Nichols, D.; Sulem, C. Editors Fields Institute Communications, Springer-Verlag, **75** (2015), 369-381.

[15] Rabelo, M. *A characterization of differential equations of type $u_{xt} = F(u, u_x, \ldots, u_{x^k})$ which describe pseudo-spherical surfaces.* An. Acad. Brasil. Ciênc. **60** (1988), 119-126.

[16] Rabelo, M. L.; *On equations which describe pseudospherical surfaces.* Stud. Appl. Math. **81** (1989), 221-248.

[17] Rabelo, M., and Tenenblat, K. *On equations of the type $u_{xt} = F(u, u_x)$ which describe pseudo-spherical surfaces.* J. Math. Phys. **29** (1990), 1400-1407

[18] Rabelo, M., and Tenenblat, K. *A classification of equations of the type $u_t = u_{xxx} + G(u, u_x, u_{xx})$ which describe pseudo-spherical surfaces.* J. Math. Phys. **33** (1992), 1044-149