Normal surfaces and polynomial computation of quantum invariants of 3-manifolds

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Resumo/Abstract:
Normal surfaces in 3-manifold triangulations are the piecewise linear equivalent of minimal surfaces, satisfying additionnal local rigidity constraints. First introduced by Kneser [2] and then studied by Haken [1], they furnish a tool to study embedded surfaces within a 3-manifold. They are at the heart of most algorithmic breakthrough in 3-manifold topology, where one usually search for the existence of a specific surface, like for *unknot recognition* [1, 3] or 3-*sphere recognition* [4].

The *quantum invariants of Turaev-Viro* [5] are a powerful family of topological invariants for distinguishing between different 3-manifolds. They are defined as state sums of combinatorial data on a 3-manifold triangulation, admit simple but slow algorithms to compute them, and are invaluable for mathematical software in geometric topology.

In this talk, we will interprete Turaev-Viro invariants in terms of normal surfaces with a bounded combinatorial area. This simple idea turns out to be the right presentation to design efficient polynomial time algorithms to compute the invariants, where instead of the existence of a certain surface one computes their number. After presenting some complexity results for the computation of Turaev-Viro invariants, in relation to counting complexity, we will present a polynomial time algorithm to compute the Turaev-Viro invariant at the 3rd root of unity, and a *fixed parameter tractable* algorithm in the first Betti number of the manifold for the Turaev-Viro invariant at the 4th root of unity. This last result is the first algorithm to use a parameter of topological nature in computational topology.

References

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