

Order preserving and order reversing operators on the class of convex functions in Banach spaces

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Abstract

A remarkable recent result by S. Artstein-Avidan and V. Milman states that, up to precomposition with affine operators, addition of affine functionals, and multiplication by positive scalars, the only *fully order preserving* mapping acting on the class of lower semicontinuous proper convex functions defined on \mathbb{R}^n is the identity operator, and the only *fully order reversing* one acting on the same set is the Fenchel conjugation. Here *fully order preserving (reversing) mappings* are understood to be those which preserve (reverse) the pointwise order among convex functions, are invertible, and such that their inverses also preserve (reverse) such order. In this paper we establish a suitable extension of these results to order preserving and order reversing operators acting on the class of lower semicontinuous proper convex functions defined on arbitrary infinite dimensional Banach spaces.

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