Seminar on $\mathbb{F}_1$-geometry

Program

(1) **Monoid schemes**
Topics: monoids with zero; ideals; localizations; monoidal spaces; spectrum; monoid schemes; base extension to $\mathbb{Z}$ and other rings; examples
References: [Dei05], [CHWW15], [Lor18a]

(2) **Relation to toric varieties**
Topics: cones; fans; toric varieties; properties of monoid schemes: irreducible, finite type, torsion free, separated, saturated; comparison theorem
References: [Ful93], [Dei08], [CHWW15]

(3) **Zeta functions** *(optional talk)*
Topics: zeta functions from counting polynomials; zeta functions for monoid schemes; functional equation
References: [Sou04], [Dei06], [Lor10]

(4) **K-theory and homotopy groups of spheres** *(optional talk)*
Topics: Barratt-Priddy-Quillen theorem; naive expectation; $+$-construction; Q-construction; $K(\mathbb{F}_1) = \pi^{\mathrm{st}}(\mathbb{S})$
References: [Sou04], [Dei06], [CLS12]

(5) **Semiring schemes**
Topics: semirings; ideals; localizations; semiringed spaces; spectrum; semiring schemes; comparison with Toën-Vaquié's approach; examples
References: [Lor17], [TV09]

(6) **Kajiwara-Payne tropicalization**
Topics: non-archimedean fields; point-wise tropicalization; bend locus; Berkovich space; restrictions of seminorms
References: [Pay09], [MS15]

(7) **Giansiracusa tropicalization**
Topics: bend relations; scheme theoretic tropicalization; universal tropicalization
References: [GG16], [GG14], [YAL17]

(8) **Ordered blue schemes**
Topics: ordered blueprints; examples; ideals; localizations; ordered blue spaces; spectrum; ordered blue schemes; examples; relation to monoid schemes and semiring schemes
References: [Lor15], [BL18], [Lor18b]

(9) **Tropicalization as a base change**
Topics: tropical hyperfield; valuations as morphisms; tropicalization as a base change; fine topology; comparison with Kajiwara-Payne tropicalization; comparison with Giansiracusa tropicalization
References: [Lor19], [Lor15]

Suggestions for the last weeks

(10) **Open problems**
List and discuss open problems

(11) **Research talks**
Invite some experts to talk on their research

(12) **Tropical ideals and balancing for scheme theoretic tropicalizations**
Topics: tropical ideals; Maclagan-Rincon weights; balancing for scheme theoretic tropicalizations
References: [MR20], [MR18], [Lor18a], [YAL17]

(13) **Moduli space of matroids**
Topics: matroids; Grassmann-Plücker functions; Proj-construction; matroid bundles; embeddings into projective space; the matroid space; the moduli property
References: [BL18], [YAL17]

(14) **Skeleta as tropical schemes**
Topics: semistable models of curves; dual graph; skeleta of the Berkovich space; blue model; dual graph as rational point set

References

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[Ful93] William Fulton. *Introduction to toric varieties*, volume 131 of *Annals of Mathematics Studies*. Princeton University Press, Princeton, NJ, 1993. The William H. Roever Lectures in Geometry.

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[Pay09] Sam Payne. Analytification is the limit of all tropicalizations. *Math. Res. Lett.*, 16(3):543–556, 2009.

[Sou04] Christophe Soulé. Les variétés sur le corps à un élément. *Mosc. Math. J.*, 4(1):217–244, 312, 2004.

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[YAL17] Lecture notes of the seminar "Tropical Scheme Theory" at YALE. Organized by Kalina Mincheva and Samuel Payne, webpage `https://web.ma.utexas.edu/users/sampayne/Math648.html`, 2017.