Exercises for Algebra II
List 1 To hand in at 28.8.2018 in class

Exercise 1.
Let P be a point in $\mathbb{R}^2$ with coordinates x and y. Show that P is constructible from a given set of points $0, 1, P_1, \ldots, P_n$ if and only if x and y are constructible (considered as points $(x, 0)$ and $(y, 0)$ of the first coordinate axis in $\mathbb{R}^2$). Conclude that the point $P_1 + P_2$ (using vector addition) is constructible from $0, 1, P_1, P_2$.

Exercise 2.
Let r be a positive real number. Show that $h = \sqrt{r}$ is constructible from 0, 1 and r.
Hint: You are allowed to use classical geometric theorems like the theorem of Thales or the theorem of Pythagoras.

Exercise 3.
Construct the following regular n-gons with ruler and compass:

1. a regular 2^r-gon for $r \geq 2$;
2. a regular 3-gon;
3. a regular 5-gon.

Exercise 4.
Prove Cardano's formula: given an equation $x^3 + px + q = 0$ with real coefficients p and q such that $\Delta = q^2/4 + p^3/27 > 0$, then

$$x = \sqrt[3]{-\frac{q}{2} + \sqrt{\Delta}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\Delta}}$$

is a solution.

Exercise 5.
Find all solutions for $x^4 - 2x^3 - 2x - 1 = 0$.
Hint: Use Ferrari's formula.

Exercise 6 (very difficult; not to hand in)**.** Find solutions to the following classical problems:

1. Given a positive real number r, is it possible to construct the cube root $\sqrt[3]{r}$?
2. Given an angle φ, is it possible to construct $\varphi/3$?
3. Given a circle with area A, is it possible to construct a square with area A?