TOPICS FOR "NONARCHIMEDEAN ANALYTIC AND TROPICAL GEOMETRY"

The first few weeks of the course will keep us busy with understanding what a tropical variety and what a Berkovich space is. So the first two chapters of the course will be:

(1) **Berkovich spaces.** This includes the following topics: non-archimedean seminorms, Banach algebras, Tate algebras, analytic functions, examples (unit discs, curves). The main reference is [3].

(2) **Tropical varieties.** This includes the following topics: amoebas, Puiseux series, bend locus, polyhedral complexes, balancing condition, examples. There are numerous introductory texts to the topic. One comprehensive source is the book [8]. Another important paper is [9].

After these first lectures, we have a good basic knowledge to enter more specific topics in the areas of tropical and NA analytic geometry. Once we have reached this point, we have to decide onwhat we want to do.

In the following, I describe some possibilities of what we could learn, but we will have to make a selection of topics. These topics are largely independent, so we could study one or two of them in arbitrary order.

(3) **Tropicalization of the moduli space of curves.** This would be study of the paper [1], including all preparatory theory, like: the classical moduli space $\mathcal{M}_{g,n}$ of curves and its compactification $\overline{\mathcal{M}}_{g,n}$, toroidal embeddings ([6]), cone complexes, stacks and stacky fans, Thuillier spaces and skeleta ([12]). An optional topic that adds well is the tropicalization of log-schemes ([13]).

(4) **Further topics in non-archimedean analytic geometry.** There are other theories of non-archimedean analytic spaces than Berkovich spaces. We could study these different approaches and compare them, which are: Tate's rigid analytic varieties ([11]), Raynaud's formal analytic geometry ([10]), Huber's adic spaces ([5]) and Fujiwara and Kato's Zariski-Riemann spaces ([4]).

(5) **Baker-Norine theory.** This theory centres around the Riemann-Roch theorem for tropical curves, and would involve the following further topics: discrete Riemann-Roch and chip firing games (as a motivation), semistable models of curves over non-archimedean fields, skeleta for strictly semistable models, tropicalization of divisors, the specialization lemma; optionally: limit linear series and Brill-Noether theory. These topics are covered in [2].

(6) **Tropical schemes.** This would cover parts of [7], and possibly some topics beyond. This includes: (ordered) blueprints, Grothendieck pretopologies and generalized scheme theory, blue schemes and semiring schemes, rational point sets, bend relations, valuations in idempotent semirings, scheme theoretic tropicalization, tropical ideals and weights.

References

[1] Dan Abramovich, Lucia Caporaso, and Sam Payne. The tropicalization of the moduli space of curves. Preprint, arXiv:1212.0373, 2012.

[2] Matt Baker, David Jensen. Degeneration of Linear Series From the Tropical Point of View and Applications. Preprint, arXiv:1504.05544, 2015

[3] Vladimir G. Berkovich. *Spectral theory and analytic geometry over non-Archimedean fields*, volume 33 of *Mathematical Surveys and Monographs*. American Mathematical Society, Providence, RI, 1990.

[4] Kazuhiro Fujiwara and Fumiharu Kato. Foundations of rigid geometry I. Preprint, arXiv:1308.4734, 2013.

[5] Roland Huber. *Etale cohomology of rigid analytic varieties and adic spaces.* Aspects of Mathematics, E30. Friedr. Vieweg and Sohn, Braunschweig, 1996.

[6] G. Kempf, F. Knudsen, D. Mumford, and B. Saint-Donat. *Toroidal embeddings. I.* Lecture Notes in Mathematics, Vol. 339, Springer-Verlag, Berlin, 1973.

[7] Oliver Lorscheid. Scheme theoretic tropicalization. Preprint, arXiv:1508.07949, 2015.

[8] Diane Maclagan and Bernd Sturmfels. *Introduction to tropical geometry*, volume 161 of *Graduate Studies in Mathematics*. American Mathematical Society, Providence, RI, 2015.

[9] Sam Payne. Analytification is the limit of all tropicalizations. *Math. Res. Lett.*, 16(3):543–556, 2009.

[10] Michel Raynaud. Géométrie analytique rigide dapres Tate, Kiehl, ... Table ronde danalyse non archimedienne. *Bull. Soc. Math. France*, Mémoire 39/40, 319–327, 1974.

[11] John Tate. Rigid analytic spaces. *Invent. Math.*, 12:257–289, 1971.

[12] Amaury Thuillier. Géométrie toroïdale et géométrie analytique non archimédienne. Application au type d'homotopie de certains schémas formels. *Manuscripta Math.*, 123(4):381–451, 2007.

[13] Martin Ulirsch. Functorial tropicalization of logarithmic schemes: the case of constant coefficients. Preprint, arXiv:1504.05544, 2013.