STUDY GROUP ON TROPICAL MODULI OF CURVES

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The intend of this reading course is to study the paper [ACP12] by Abramovich, Caporaso and Payne on the tropicalization of the moduli space of curves. This includes introductions to toroidal embeddings, cone complexes, Berkovich spaces and Thuillier's retraction to the skeleton, as well as a review of Deligne-Mumford stacks and $\overline{\mathcal{M}}_{g,n}$. Moreover, we take advantage to explore related topics as skeleta of Berkovich spaces given by semistable models and Tyomkin's correspondence theorems.

We divide the reading course into the following 5 topics:

(1) Introduction to Berkovich spaces, toroidal embeddings, cone complexes and Thuillier's retraction to the skeleton. This follows [ACP12], using additional material from [Ber09], [Thu07]. *Speaker: Oliver*
(2) Skeleta from semistable models and their relation to tropicalizations. This summarizes [BPR13] and [Wer15]. *Speaker: Ethan*
(3) Introduction to the moduli space of (tropical) curves. Extensions of the earlier results to Deligne-Mumford stacks. This follows [ACP12], summarizing some results from [CT07], [Cos10] and [HM98]. *Speaker: open*
(4) The Berkovich analytification of the moduli space of curves and its tropicalization. This is the heart piece of [ACP12]. *Speaker: open*
(5) Tyomkin's correspondence theorems between rational curves in projective space satisfying point conditions and their tropical counterparts. This is [Tyo15]. *Speaker: open*

We will meet for the rest of this semester every **Monday for 13:30–15:30 in room 345 at IMPA**, beginning with **October 19**.

References

[ACP12] Dan Abramovich, Lucia Caporaso, and Sam Payne. The tropicalization of the moduli space of curves. Preprint, arXiv:1212.0373, 2012.
[ACG11] E. Arbarello, M. Cornalba, P. Griffiths. Geometry of Algebraic curves, volume II. Grundlehren der mathematishen Wissenschaften, Vol 268, Springer, Heidelberg, 2011.
[BPR13] Matthew Baker, Sam Payne, and Joseph Rabinoff. On the structure of non-Archimedean analytic curves. In *Tropical and non-Archimedean geometry*, volume 605 of *Contemp. Math.*, pages 93–121. Amer. Math. Soc., Providence, RI, 2013.
[Ber09] Vladimir Berkovich. Non-archimedean analytic spaces. Lecture notes, http://www.wisdom.weizmann.ac.il/~vova/Trieste_2009.pdf, 2009.
[CT07] B. Conrad and M. Temkin. Non-Archimedean analytification of algebraic spaces. *J. Algebraic Geom.* 18(4):731–788, 2009.
[Cos10] Izzet Coskun. Birational geometry of moduli spaces. Lecture notes, http://homepages.math.uic.edu/~coskun/utah-notes.pdf, 2010.
[HM98] J. Harris and I. Morrison. Moduli of curve. Graduate Texts in Mathematics, vol. 187, Springer-Verlag, New York, 1998.
[Thu07] Amaury Thuillier. Géométrie toroïdale et géométrie analytique non archimédienne. Application au type d'homotopie de certains schémas formels. *Manuscripta Math.*, 123(4):381–451, 2007.
[Tyo15] Ilya Tyomkin. Enumeration of rational curves with cross-ratio constraints. Preprint, arXiv:1509.07453, 2015.
[Wer15] Annette Werner. Analytification and tropicalization over non-Archimedean fields. Preprint, arXiv:1506.04846, 2015.