Exercises for Algebra IIInstituto Nacional de Matemática Pura e AplicadaSeries 8Oliver Lorscheid (professor)To hand in at 17.10.2014 in the exercise classJosé Ramón Madrid Padilla (monitor)

Exercise 1.

Find a normal basis for the splitting field L of $f = T^3 - 11$ over \mathbb{Q} . Derive from this a basis of E/\mathbb{Q} for every subfield E of L.

Exercise 2.

Are the roots of the following polynomials construcible over \mathbb{Q} ?

1. $f_1 = T^4 - 2$ 2. $f_2 = T^4 - T$ 3. $f_3 = T^4 - 2T$

Exercise 3.

Let K be a subfield of \mathbb{C} and a a root of $T^2 - b \in K[T]$. Show that every element of K(a) is constructible over K. Use this to explain the relationship between the two definitions of constructible numbers from sections 1.1 and 4.7 of the lecture.

Exercise 4.

Construct a regular 5-gon over \mathbb{Q} with ruler and compass.