Exercises for Algebra II Series 5 To hand in at 19.9.2014 in class

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Exercise 1.
Calculate the Galois groups of the splitting fields of the following polynomials over $\mathbb{Q}$.

1. $f_1 = T^3 - 1$;
2. $f_2 = T^3 - 2$;
3. $f_3 = T^3 + T^2 - 2T - 1$.

Hint: If ζ is a 7-th root of 1 (different from 1), then $\zeta^i + \zeta^{7-i}$ is a root of f_3 for $i = 1, 2, 3$.

Exercise 2.
Show that $\mathbb{Q}[\sqrt{2}, i]/\mathbb{Q}$ is Galois. What is the Galois group? Describe (again) all intermediate extensions of $\mathbb{Q}[\sqrt{2}, i]/\mathbb{Q}$.

Exercise 3.
Let L be the splitting field of $f = T^4 - 3$ over $\mathbb{Q}$. What is the Galois group of $L/\mathbb{Q}$? Make a diagram of all subgroups of $\mathrm{Gal}(L/K)$ that illustrates which subgroups are contained in others.

Exercise 4.

1. Find a finite separable (but not normal) field extension L/K that does not satisfy the Galois correspondence.
2. Find a finite normal (but not separable) field extension L/K that does not satisfy the Galois correspondence.
3. **(Bonus)** Find a normal and separable (but not finite) field extension L/K that does not satisfy the Galois correspondence.