
Exercise 1.
Let $F = K(T)$ be a rational function field over K. Show that $\mathrm{Cl}^0 F$ is the trivial group.

Exercise 2.
Let F/K be a function field of dimension 1.

1. Let D a divisor and $P = 1 \cdot v$ a prime divisor. Show that $f/g \in \mathcal{O}_v$ if $f, g \in L(D+P)$, but $g \notin L(D)$.
2. Conclude that $l(D+P) \leq l(D) + \deg P$.
3. Show that $l(D)$ is finite for every divisor D.

Exercise 3.
Let F/K be a function field of dimension 1. Show that for every divisor D, the number of effective divisors in $[D] \in \mathrm{Cl}F$ is $(q^{l(D)} - 1)/(q-1)$.

Exercise 4. Let F be the fraction field of $A = \mathbb{F}_3[X,Y]/(Y^2 - X^3 + X)$, which is a function field of dimension 1 over $\mathbb{F}_3$. Show that $\mathrm{Cl}^0 F$ has precisely 4 elements.

Hint: Use without proof that the genus of F is $g = 1$. Then use the result of the previous exercise, combined with the arguments of the proof for the finiteness of the class number, to establish a relationship between $\mathrm{Cl}^0(F)$ and the places of degree 1 of F. Finally use the inclusion $\mathbb{F}_3[X] \subset A$ to show that the places of F whose restriction to $\mathbb{F}_3(X)$ is not v_∞ correspond to the homomorphisms $A \to \mathbb{F}_3$, while v_∞ has only one extension to F. Then count the homomorphisms $A \to \mathbb{F}_3$, i.e. solutions of $Y^2 - X^3 + X = 0$ with $X, Y \in \mathbb{F}_3$.