Rough singular integrals and the maximal function: new borderline weighted estimates

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Resumo/Abstract:
Muckenhoupt-Wheeden [MW] in the seventies and Sawyer [S] in the eightees, established some one-dimensional highly nontrivial extensions of the weak type $(1,1)$ property of the maximal function involving weights. These results were conjectured to hold for the Hilbert transform and for the maximal function in higher extensions. In the first part of this lecture we will survey about these conjectures that were proved and extended in different directions in [CMP], [OP] and [OPR]. Then we will discuss about the main open conjecture that has been recently settled in [LOP].

In the second part of this lecture we will discuss a recent work [LPRR] where we solved some conjectures for rough singular integrals and weights. The link of these two parts of the lecture is the classical good-lambda estimate between Calderón-Zygmund operators and the maximal functions due to R. Coifman and C. Fefferman leading to some strong or weak L^p estimates with A_∞ weights. We will show that a corresponding result holds for rough singular integrals and the Bochner-Riesz operator at critical level even though there is no such as good-λ in the literature. This result is key in the solution of the above mentioned conjectures combined with some extrapolation theorems for the class A_∞ [CGMP] together with a sparse formula found by Conde-Culiuc-Di Plinio-Ou. Further quite sharp qualitative and quantitative results will be presented.

References

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