On a Schrödinger-Born-Infeld system

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Resumo/Abstract:
We are interested in the following Schrödinger-Born-Infeld system

$$
\begin{cases}
-\Delta u + u + \phi u = |u|^{p-1}u & \text{in } \mathbb{R}^3, \\[2em]
-\operatorname{div}\left(\dfrac{\nabla \phi}{\sqrt{1 - |\nabla \phi|^2}} \right) = u^2 & \text{in } \mathbb{R}^3,
\end{cases}
$$

where $p \in [5/2, 5)$ is given and $u, \phi : \mathbb{R}^3 \to \mathbb{R}$ are the unknowns. The system appears when we look for solitary waves for the Schrödinger equation coupled with an electromagnetic field theory which generalizes the classical Maxwell theory. This new theory of electromagnetism was proposed by Born-Infeld in order to overcome the infinite energy problem associated with a point-charge source in the original Maxwell theory. Indeed the second equation of the system is not the usual Gauss law for the electric field, i.e. $-\Delta \phi = u^2$, but it takes into account the modified Lagrangian of the Born-Infeld electromagnetic theory (see [1, 2, 3, 4]).

The existence of a solution for the system is proved by using variational methods.

References

[1] M. Born, *Modified field equations with a finite radius of the electron,* Nature **132** (1933) 282.

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[3] M. Born, L. Infeld, *Foundation of the new field theory,* Nature **132** (1933) 1004.

[4] M. Born, L. Infeld, *Foundation of the new field theory,* Proc. Roy. Soc. A **144** (1934) 425–451.