Lagrangian structure for non-barotropic compressible fluids in two dimensions

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Abstract

In this work we study the Lagrangian structure for weak solutions of Navier-Stokes equations for non-barotropic compressible fluids in dimension two, i.e., we prove the uniqueness of particle trajectories for two-dimensional compressible fluids, including the energy equation (non-isentropic fluids). Our result extends to non-barotropic fluids the corresponding result in [3] for barotropic fluids, when the spatial dimensio is two .

In order to prove our main result (uniqueness of particle trajectories) we need to extend some estimates presented in [2] for the two dimensional case. We use techniques presented in [3] and [4] to get L^2 estimates for the material derivative of the velocity field of the fluid and for the specific internal energy, as well. Besides, we need the classical energy estimates and pointwise bounds for density and specific internal energy. It is also proved, via compactness arguments, that those estimates, obtained a priori for approximated solutions, hold for the weak solutions. The main assumptions, as in several papers by David Hoff and some others, are that the initial energy is small and the initial velocity belongs to a positive Sobolev space. Besides, we also assume that the initial density is strictly positive, i.e. greater than some positive constant (in particular, no vacuum is allowed).

The main difficulty to obtain the needed estimates in the non-barotropic (non-isetropic) case, in comparison with the barotropic (isentropic) case, is due to the dependence of the pressure function on the temperature. We

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overcome this difficulty by using the energy equation and improving some ideias from [3] and [4].
Our result of existence of weak solutions extends some previous results for two dimensions. First, we slightly extend some results in [2] by proving appropriate estimates that ensure the Lagrangian structure. Secondly, we extend some results in [3] to the non-barotropic case.

More general solutions to the Navier-Stokes system, with large energy, but less regular than the one considered, can be found in [1]. However, for this broad class of solutions the Lagrangian structure is not clear, due to the lack of regularity properties to the solutions.

Some references:

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[4] Xiangdi Huang, Jing Li, and Zhouping Xin. Global well-posedness of classical solutions with large oscillations and vacuum to the three-dimensional isentropic compressible navier-stokes equations. *Comm. Pure Appl. Math.*, 65 (4):549–585, 2012.