Zero Range Process with Sitewise Disorder: XVII Escola Brasileira de Probabilidade, Mambucaba, 9 August 2013

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The Model

Let $\bar{\mathbf{N}} := \mathbf{N} \cup \{\infty\}$ and let $\mathbf{X} = \{\bar{\mathbf{N}}\}^{\mathbf{Z}}$ be the state space of the process. Let $c > 0$ and define the set of admissible environments as $\mathbf{A} = (c, 1]^{\mathbf{Z}}$ and let $\alpha(x), x \in \mathbf{Z}$ be a stationary ergodic sequence with marginal $\mathbf{Q}$. We assume that c is the infimum of support of $\mathbf{Q}$. Let $g : \mathbf{N} \to [0, 1]$ be a nondecreasing function with $0 = g(0) < g(1) \leq \lim_{k\to\infty} g(k) := g(\infty) = 1$. Let $1/2 < p \leq 1$ and $q = 1 - p$.
For $\eta \in \mathbf{X}$ and any local function f left the quenched Markov process $(\eta_t^\alpha), t \geq 0$ on $\mathbf{X}$ be defined by the generator

$$L^\alpha f(\eta) = \sum_{x\in\mathbf{Z}} \alpha(x) \left[pg(x)(f(\eta^{x,x+1}) - f(\eta)) + qg(x)(f(\eta^{x-1,x}) - f(\eta))\right]$$

The invariant measures

Let

$$\theta_\lambda(n) := Z(\lambda)^{-1} \frac{\lambda^n}{g(n)!}, \quad n \in \mathbb{N}$$

where $g(n)! = \prod_{k=1}^{n} g(k)$ for $n \geq 1$, $g(0)! = 1$, and $Z(\lambda)$ is a normalizing factor. For $\lambda \leq c$, we denote by μ_λ^α the invariant measure of L^α defined as the product measure on $\mathbf{X}$ with one-site marginal $\theta_{\lambda/\alpha(x)}$

Let

$$R(\lambda) := \sum_{n=0}^{+\infty} n\theta_\lambda(n) \tag{1}$$

denote the mean value of θ_λ. The quenched mean particle density at x under μ_λ^α is defined by

$$R^\alpha(x,\lambda) = \mathbb{E}_{\mu_\lambda^\alpha}[\eta(x)] = R\left(\frac{\lambda}{\alpha(x)}\right) \tag{2}$$

The annealed mean particle density (independent of x) is given by

$$\overline{R}(\lambda) := \int_{(c,1]} R^\alpha(x,\lambda)Q(d\alpha) \tag{3}$$

We make the assumption

$$\rho_c := \overline{R}(c) < +\infty \tag{4}$$

we define a change of parameter to index invariant measures by their mean density instead of chemical potential λ. To this end, let $\rho \mapsto \Lambda(\rho)$ be the inverse of $\lambda \mapsto \overline{R}(\lambda)$, which is a C^∞ increasing function defined on $[0, \rho_c]$. we define the macroscopic flux function as the equilibrium expectation of the microscopic flux function (which is thus a C^∞ increasing function on $[0, \rho_c]$)

$$\begin{aligned} f(\rho) &:= \int_{\mathbf{X}} [p\alpha(x)g(\eta(x)) - q\alpha(x+1)g(\eta(x+1))]\, d\nu_\rho^\alpha(\eta) \\ &= (p-q)\Lambda(\rho), \quad \rho \in [0, \rho_c] \end{aligned} \tag{5}$$

We make the following assumption:

(H) The concave envelope of f is not affine on any interval of the form $(\rho_c - \varepsilon, \rho_c)$ with $\varepsilon > 0$.

Previous Results

Benjamini, Ferrari and Landim [bfl 96] considered an asymmetric simple exclusion process where each particle has a random jump rate and the corresponding zero range process with $g(\eta) = 1\{\eta > 0\}$. Under the same condition on the site-wise disorder they proved the existence of a critical density ρ_c above which there were no product invariant measures for the above zero range process and also proved quenched hydrodynamics in the subcritical regime. Andjel, Ferrari, Guiol and Landim [afgl 2000] proved for the totally asymmetric zero range process with site-wise disorder and $g(\eta) = 1\{\eta > 0\}$ the following: almost every initial configuration with lower left empirical density greater than ρ_c converges to as time goes to infinity to the upper invariant measure with density ρ_c.

Main Theorem

Theorem

Let $\eta_0 \in \mathbb{N}^{\mathbb{Z}}$ be such that

$$\liminf_{n\to\infty} n^{-1} \sum_{x=-n}^{0} \eta_0(x) \geq \rho_c \tag{6}$$

Then for $\mathcal{P}$-a.e. environment $\alpha \in \mathbf{A}$, the quenched process $(\eta_t^\alpha)_{t\geq 0}$ with initial state η_0 converges in distribution to μ_c^α as $t \to \infty$.

Proof, Ingredients

Upper bound is obtained modifying the argument used in [afgl];
Main ingredients in the proof of lower bound are 1) flux estimates based on initial configurations; 2) Hydrodynamics for a semi infinite system with source/sink to the left of origin (or x_0 in general); 3) Derivation of local equilibrium.
Work in progress (to be written): Necessity of the lower limit condition and counterexample without nearest neighbor condition.

Thank You!