ON THE STABILITY OF INITIAL CONDITIONS FOR A PARTICULAR NONLINEAR PARABOLIC PROBLEM

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Abstract: This work studies initial conditions for a classical quasi-linear parabolic equation with exponential nonlinearity such that blow-up is developed. This is a very old question in parabolic PDE's, as for example it corresponds to the question of blow-up develop in Thermal Runaway. Very define criteria are given for classifying initial condition that develop blow-up.

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1 PRESENTATION

This work studies a particular case of the very classical and old problem: given the quasi linear parabolic problem

$$\begin{aligned} \frac{\partial v}{\partial t} &= \Delta v + \lambda\, f(v) \quad \text{in }]0,T) \times \Omega, \qquad (1)\\ v(0,x) &= v_0(x) \quad \text{in } \Omega, \\ v &= 0 \quad \text{on }]0,T) \times \partial\,\Omega, \end{aligned}$$

for which initial conditions v_0 the solution of (1) develops blow-up?, this is there exists a time T (finite or not), a point $x \in \Omega$ and a nontrivial sequence $\{(t_i, x_i)\}_i \subset]0,T[\times\Omega$ with $t_i \nearrow T$ increasingly, $x_i \nearrow x$, and such that the solution of (1) with initial condition v_0 satisfies $|v(t_i, x_i)| \nearrow \infty$. As it is classical, Ω is open, bounded and with regular boundary set in $I\!R^n$, while $f : I\!R \longrightarrow I\!R$ is at least a continuous function.

Previous question has a long date as can be seen, for example, in the theory of *Thermal Runaway*, where blow-up is interpreted as a very fast reaction which cannot balance its temperature. In a different application, the same question has enormous significance in the *Navier-Stokes* equation, where its answer is associated to the development of Turbulence.

The case of (1) with exponential nonlinearity $f(t) = e^t$, known as the *Gelfand Problem*, is the main focus of the work. This problem has many application in both theoretical and applied mathematics through *Differential Geometry* and *Thermal Runaway Theory* respectively.

A short review of classical literature on the Gelfand Problem, mainly chapter three of reference **[2]** with its sections entitled *Blowup: When?*, *Blowup: Where?* and *Blowup: How?*, shows that despite of having a large number of results in the three previously mentioned questions, it is not available an answer to the posed question at the beginning of this note. The situation is not very different in most of the literature that studies quasi linear parabolic equations, where not many results that give answer to the posed question are available, in comparison to other topics concerning (1); see for example **[3]**, where stability theorems for solutions of (1) are given in the last chapter.

By exploiting the global behaviour of the real function $g(x) = x\,e^x$ in the whole real axis, it is given a very defined criterion for the posed question, which up to now lacks of a concrete answer in the literature. Surprisingly, the use of previous function g is inspired from the basic result of existence of exponential solutions for the first order linear delay equation, see **[10]**. Some numerics supporting the results are presented too, in simple geometries. Finally, a review on the Gelfand Problem with *Neumann* homogeneous boundary condition is done.

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