

**PROPOSAL FOR A MINICOURSE AT IMPA
SUMMER PROGRAM 2016**

**SYMMETRIES AND BICATEGORIES IN NONCOMMUTATIVE
GEOMETRY**

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Noncommutative geometry is supposed to deal with badly behaved spaces by replacing them by C^* -algebras. The C^* -algebras are usually created via a two-step procedure: one first assigns a groupoid suitably attached to the situation in hand (usually associated with some sort of combinatorial/dynamical data or some equivalence relation). In many situations these groupoids come with canonical "symmetries" (given by some sort of action of a (semi)group or groupoid) and it is natural to ask whether these symmetries lift to the corresponding C^* -algebras. One of the goals of this series of lectures is to explain how this can be achieved. This will mainly follow the paper [2].

Sometimes, one is forced to allow groupoids whose arrow space has singularities. Such situations may be handled using bigroupoids. In particular, we describe how to desingularise locally Hausdorff groupoids to Hausdorff bigroupoids. Bigroupoids allow to integrate arbitrary Lie algebroids, as shown by Tseng and Zhu [7], overcoming the obstruction found by Crainic and Fernandes [5]. Bigroupoids contain crossed modules of groupoids as special cases, whose actions on C^* -algebras were studied in [3]. I will explain how bigroupoids act on groupoids and C^* -algebras, extending work in [1–4], and define crossed products for such actions. The relevant notion of actions for higher groupoids was introduced in the thesis of Li Du [6]. I will explain recent work about a universal property that characterises these C^* -algebras and which is new even for ordinary locally compact groupoids.

Duration of the minicourse: approx. 8 hours (5 lectures).

REFERENCES

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