WOLBACHIA INFESTATION OF AEDES POPULATIONS BY FEEDBACK: ALMOST GLOBAL STABILIZATION RESULT FOR A BISTABLE SYSTEM

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Abstract

The use of bacteria Wolbachia is a promising method presently considered to block the transmission of dengue and chikungunya viruses. Systematic procedures for introduction of mosquitos infected by the bacteria in a healthy population are still to be studied. This is a central question, with heavy impact on the cost and efficiency.

The aim of the present study is the synthesis of a method allowing the reduction of the number of introduced mosquitos, and consequently the cost, without putting at risk the success of the infestation (something that could happen e.g. if the initial size of the population has been underestimated). Using the fact that measurements are completed during the whole release process, techniques from the theory of control of dynamical systems are used to define the quantity to be introduced. The original system is shown to have two stable equilibria, corresponding to Wolbachia-free and complete infestation situations. A simple feedback law is proposed and shown to have the capacity to asymptotically settle the bacteria. Up to our knowledge, this is the first attempt to use feedback for introduction of Wolbachia within a population of arthropods.

The techniques are based on the theory of monotone systems [2, 3], recently extended to analyze the asymptotic behavior of input-output monotone systems

closed by negative feedback [1, 4, 7]. Due to bistability, the considered input-output system has multivalued static characteristics, but the existing results [5, 6] are unable to prove almost-global stabilization, so ad hoc analysis has to be used.

References

[1] J.-L. Gouzé, A criterion of global convergence to equilibrium for differential systems with an application to Lotka-Volterra systems, *Inria research report RR-0894* (1988)

[2] M.W. Hirsch, Stability and convergence in strongly monotone dynamical systems. *J. reine angew. Math* **383** (1988) pp. 1–53

[3] H.L. Smith, *Monotone dynamical systems: an introduction to the theory of competitive and cooperative systems*, Mathematical surveys and monographs 41, American Mathematical Society (1995)

[4] D. Angeli and E.D. Sontag, Monotone control systems, *IEEE Transactions on Automatic Control* **48** (2003) pp. 1684–1698

[5] M. Malisoff and P.D. Leenheer, A small-gain theorem for monotone systems with multi-valued input- state characteristics, *IEEE Transactions on Automatic Control* **41** no. 2 (2006) pp. 287–292

[6] T. Gedeon and G. Hines, Multi-valued characteristics and Morse decompositions. *Journal of Differential Equations* **247** no. 4 (2009) pp. 1013–1042

[7] G. Enciso, Fixed points and convergence in monotone systems under positive or negative feedback, *International Journal of Control* **87** no. 2 (2014), pp. 301–311