DYNAMICAL GAME INTERACTIONS ON NETWORKED POPULATIONS

Chiara Mocenni[1] and **Dario Madeo**[1]
[1]Department of Information Engineering and Mathematics
University of Siena
Via Roma, 56 - 53100, Siena (Italy)
mocenni@dii.unisi.it

Abstract

The mathematical formulation of the dynamical game interactions of a finite set of players organized on an arbitrary graph is presented. Classical results on multipopulation evolutionary game theory are used in combination with graph theory to obtain the equations. Specifically, the players, located at the vertices of the graph, are interpreted as subpopulations of a multipopulation dynamical game. The members of each subpopulation are replicators, engaged at each time instant into 2-player games with the members of other connected subpopulations. The obtained equation does not require any assumption on the game payoff matrices nor graph topology. The equation is the following:

$$\dot{x}_{v,s} = x_{v,s}(p^{\mathcal{G}}_{v,s} - \phi^{\mathcal{G}}_v),$$

where $x_{v,s}$ represents the share of strategy s in vertex v, $p^{\mathcal{G}}_{v,s}$ is the payoff earned by vertex v using the pure strategy s and $\phi^{\mathcal{G}}_v$ is the average payoff of vertex v. The payoff $p^{\mathcal{G}}_{v,s}$ is defined as:

$$p^{\mathcal{G}}_{v,s} = \boldsymbol{e}_s^T \boldsymbol{B}_v \boldsymbol{k}_v,$$

and the average payoff $\phi^{\mathcal{G}}_v$ is:

$$\phi^{\mathcal{G}}_v = \boldsymbol{x}_v^T \boldsymbol{B}_v \boldsymbol{k}_v,$$

where

$$\boldsymbol{k}_v = \sum_{w=1}^{N} a_{v,w} \boldsymbol{x}_w.$$

The term $\boldsymbol{k}_v$ accounts for the graph topology through the adjacency matrix $\boldsymbol{A} = \{a_{v,w}\}$ of the graph $\mathcal{G}$.

The dependence of the stability of steady states and Nash equilibria on the payoff matrices and graph topology are discussed. Extended simulations have been performed to show the resulting complex dynamics.

References

[1] J. Hofbauer and K. Sigmund, "Evolutionary game dynamics", *Bull. Am. Math. Soc.*, vol. 40, no. 4, pp. 479-519, 2013.
[2] G. Szabó and C. Hauert, "Evolutionary prisoner's dilemma games with voluntary participation", *Phys. Rev. E*, vol. 66, pp. 062903, 2002.
[3] J.W. Weibull, *Evolutionary Game Theory.* Cambridge, MA: MIT Press, 1995.
[4] D. Madeo, L.R. Comolli and C. Mocenni, "Emergence of Microbial Networks as Response to Hostile Environments", *Front. Microbiol.*, vol. 5, no. 407, doi:10.3389/fmicb.2014.00407, 2014.
[5] D. Madeo and C. Mocenni, "Game interactions and dynamics on networked populations", *IEEE Trans. Aut. Contr.*, doi:10.1109/TAC.2014.2384755, 2014.