AN INTRODUCTION TO GENERAL RELATIVITY

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Pre-requisites: Familiarity with the first part of the "Riemannian Geometry" class at IMPA (from Riemannian metrics to the Codazzi equations) is assumed. Having some background on the (linear) wave and Laplace equations is helpful. Familiarity with classical physics is *not* required but helpful.

Summary: This class is aimed at introducing the theory of General Relativity. Instead of postulating the theory, we begin by discussing Einstein's considerations that first led to Special Relativity and then to General Relativity which necessarily must contain a theory of gravity, accomplished by the Einstein equations. We study some basic properties of the Einstein equations and present special solutions central to physics. We finish the class by discussing two advanced topics, the Cauchy problem for the Einstein equations and the famous "Positive Mass Theorem". In detail, the content of the class is the following:

I. **A Quick Review of Pre-Relativistic Physics** (1.5 lectures) [7, 1]
   - Newtonian equation of motion; Galilean transformation and Galilean Principle of Relativity.
   - Maxwell's equations; speed of light; Maxwell's equations under Galilean transformations; aether theory.
   - The crisis of 19-th century physics: observations of light speed and Michelson-Morley experiment "killing" the aether theory.
   - Lorentz Transformations.

II. **Special Relativity** (3.5 lectures) [7, 3, 1]
   - The axioms of Special Relativity and their physical motivation.
   - Time dilatation; length contraction; proper time and length.
   - Relativistic equations of motion; Minkowski force; $E = mc^2$.
   - Space and time as a unity: Minkowski spacetime.
   - Energy-momentum tensor; example for fluids and electromagnetism.

III. **General Relativity and Einstein Equations** (5 lectures) [7, 3, 1]
   - Why accelerated coordinate transformations invoke gravity.
   - Axioms of General Relativity and their motivation.
   - Basics about Lorentz geometry; the light cone.
   - The Einstein equations: describing gravity as spacetime curvature.
   - The geodesic deviation equation for particle acceleration.
   - Linearized Einstein equations; gravitational waves; Newtonian limit.

IV. **Special Solutions of the Einstein Equations** (4 lectures) [4, 7, 8]
   - Minkowski spacetime; Penrose diagrams.

- FRLW-metric: Cosmology, the Big Bang and Dark Energy.
- Black holes: Schwarzschild, Reisner-Nordström and Kerr metrics.
- Formation of black holes by gravitational collapse of fluids.

V. **Cauchy Problem for the Einstein Equations** (5 lectures) [7, 1, 5]
- Causal Structures; globally hyperbolic spacetimes; foliations.
- The Cauchy problem for Maxwell's equations.
- Formulation of Cauchy problem for Einstein equations; constraint and evolution equations.
- The wave gauge and the reduced Einstein equations.
- Propagation of wave gauge for vacuum Einstein equations and local existence (applying Leray's theorem).
- Solving Einstein equations with a matter source. (If time permits.)

VI. **The Mass of Spacetime** (5 lectures) [7, 2, 6]
- Physical motivation for assigning mass to spacetime.
- Asymptotic flatness; notions for mass; ADM mass.
- Positive Mass Theorem (assuming existence of minimal surfaces).

References

[1] Y. Choquet-Bruhat, *General Relativity and the Einstein Equations*, Oxford University Press, 2009.
[2] D. Christodoulou, *Mathematical Problems of General Relativity*, European Mathematical Society, Zurich Lectures in Advanced Mathematics, 2008.
[3] A. Einstein, *Relativity: The Special and General Theory*, Methuen & Co. Ltd., 1920.
[4] S.W. Hawking and G.F.R. Ellis, *The Large Scale Structure of Spacetime*, Cambridge University Press, 1973.
[5] H. Ringström, *The Cauchy Problem in General Relativity*, European Mathematical Society, 2009.
[6] R. Schoen, S.T. Yau, "On the proof of the positive mass conjecture in general relativity", Comm. Math. Phys., **65**, (1979). 45-76.
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[8] S. Weinberg, *Gravitation and Cosmology*, John Wiley & Sons, New York, 1972.