

Bank Runs and Risk-Shifting in the Presence of Information Disclosure Lags

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Abstract

I study the effects of information disclosure lags on incentive provision and runs for a bank financed with short-term debt. Runs occur because of intertemporal coordination problems and bank management has discretion over the asset riskiness. Creditors receive lagged information about the asset's value. A disclosure lag balances the benefits of reduced risk-shifting with the cost of runs when the bank's assets have low-growth rates. When the bank's assets have high growth opportunities, a disclosure lag increases the occurrence of runs and hence decreases the value of the bank. High-growth assets decrease the manager's risk appetite and this translates into a reduced run probability.

1 Introduction

Emergency lending facilities like the discount window became an important tool used by the Federal Reserve during the financial crisis. The Fed encourages institutions to borrow from emergency funding facilities by keeping confidential some of the participants' information. Confidential borrowing by financial institutions has both social benefits and costs. Confidential borrowing costs include worsening of risk-taking activities by the bank manager while benefits include the control of bank runs. Given the costs and benefits of confidentiality, it remains unclear to what extent there should be detailed disclosure of information about financial institutions that use emergency

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lending facilities. I study the effects that information disclosure of financial institutions has on the occurrence of bank runs and manager risk-taking activities. A balance between the occurrence of bank runs and risk-taking is achieved when information is disclosed with a lag. A disclosure lag policy is studied as a tool that trade-offs the costs and benefits of disclosure.

Benefits of information disclosure include improved market discipline, allowing creditors to take corrective action by restricting the money deposits or the banks flow of funding before the bank fails. In particular, this restriction of money deposits can serve as a control device for bank managerial moral hazard and risk shifting¹. For example, bank managerial risk-taking activities could diminish by the disclosure of banks' stress tests. Stress test evaluate whether large banks have sufficient capital to absorb losses resulting from adverse economic conditions. The appropriate amount of capital concerns bank creditors and it is helpful in evaluating manager performance.

On the other hand, as presented by Goldstein and Sapra (2014), costs associated with disclosure may include the impact on ex-post actions of bank creditors. For example, if a bank creditor has strategic concerns about the other creditors' actions, their behavior could fail to coordinate in an efficient way after information disclosure, causing a bank run. Bank runs have been researched as a leading cause of the financial crisis 2007-2009². The capital structure in the shadow banking system was prone to inefficient runs due to a coordination problem ultimately introduced by short-term debt that was similar in characteristics and function to demand deposits.

The costs and benefits of information disclosure can be affected by the way in which information is transmitted to market participants. Disclosure often occurs with a time lag after the event, but there is still a debate on what should be an appropriate lag or if there should be a lag at all for information disclosure. For instance, the Dodd-Frank Financial Reform Act of 2010 requires discount window participants to be disclosed with a two year lag. This was a change from a non-disclosure policy that the Fed had since the creation of the discount window in 1913. The lag in

¹See, for example, Calomiris and Kahn (1991), Jensen and Meckling (1976), Kashyap, Rajan and Stein (2008) and Diamond and Rajan (2000 and 2001).

²See He and Xiong (2012), Morris and Shin (2009), Acharya (2010), Gorton and Metrick (2011), and Brunnermeier (2009).

information disclosure is seen as a way to balance the benefit of market discipline while mitigating the cost of a bank run³.

I study the effects of disclosure lags on runs and incentive provision for a bank financed with short-term debt. In the model, runs occur because there is an intertemporal coordination problem among creditors as in He and Xiong (2012). Also, there is a risk-shifting problem that allows bank management to have discretion over the asset riskiness. I model the information disclosure lag explicitly, where creditors receive delayed information about the asset's value. My contribution is threefold. First, I show that when the bank holds assets with low growth rates, a disclosure lag is beneficial because it balances the benefits of reduced risk-shifting with the cost of runs. A disclosure lag increases bank value because the bank manager reduces risk as creditors get nervous with opacity. The risk reduction effect dominates and the bank value increases as the bank run probability decreases.

Second, when the bank's assets have high growth rates, a disclosure lag increases the probability of bank runs and hence decreases the value of the bank. This occurs because in the presence of a disclosure lag, the manager increases asset risk taking advantage of creditors. The increase in asset risk dominates and decreases bank value. Third, assets with high growth rates decrease the manager's appetite for risk and this translates into a reduced run probability and a higher bank value. This occurs because high growth rate assets decrease risk taking by the manager and creditors run less frequently when assets are safe.

These contributions are presented in by incorporating three key components in a model. First, I model a risk-shifting problem in which the bank manager chooses the riskiness of assets to maximize her utility. The bank raises funds in the form of short-term debt and uses the resources to finance the assets. Without debt financing, the bank value is independent of the asset risk. However, in the presence of debt financing, the bank manager has incentives to risk-shift when the asset value is low.

³See Fed Releases Discount-Window Loan Records Under Court Order. <http://www.bloomberg.com/news/articles/2011-03-31/federal-reserve-releases-discount-window-loan-records-under-court-order>

The second component of the model is a coordination problem among creditors. Bank runs occur when creditors decide not to rollover the debt. The bank finances assets by raising funds in the form of staggered short-term debt. As debt matures, creditors must decide whether to rollover or withdraw their funds. The staggered short-term debt creates an intertemporal coordination problem that causes bank runs as in He and Xiong (2012). If the bank cannot sustain the run, it is forced to liquidate the assets inefficiently. The coordination problem is made worse when assets are risky, increasing the probability of a run.

The third component of the model is the structure of information disclosure to market participants. Bank creditors receive lagged information about the current value of the bank's asset. Creditors are aware of the fact that they are receiving outdated information and form rational expectations about the value of the asset today. Creditors' response to a disclosure lag is a function of the project's growth-to-risk ratio. Creditors run less frequently with a disclosure lag if the project has a high growth-to-risk ratio. The reason is that low realizations of the project's value observed with a lag are expected to recover given the high growth rate. On the contrary, creditors' incentives to run increase when the asset has a low growth-to-risk ratio.

A lag in information disclosure also affects the optimal behavior for the bank manager however. The manager's payoff is equal to a call option on the assets of the bank for which time to maturity is equal to the disclosure lag. A longer disclosure lag makes risk shifting more attractive because it increases the marginal value of risk.

When the bank holds assets with low-growth rates a disclosure lag balances the benefits of reduced risk-shifting with the cost of bank runs. Low-growth rate assets motivate the bank manager to increase her payoff by risk-shifting to the risky project. This produces an asset with a low growth-to-risk ratio. As the disclosure lag increases, creditors optimal decision is to run more frequently because of the low growth-to-risk ratio of the project. Seeing this, the manager optimally reduces risk in order to reduce the probability a bank run. The risk reduction effect dominates the nervousness of creditors and the bank value increases with a disclosure lag.

When the bank's assets have high growth rates, a disclosure lag increases the occurrence of

runs and hence decreases the value of the bank. High growth assets prevent excessive risk taking by the manager. With low-risk taking, the project has a high growth-to-risk ratio. As the disclosure lag increases, creditors' incentive to run decreases given the good quality of the project. The bank manager sees that the run probability decreases and takes advantage of this by increasing risk. The increase in asset risk effect dominates and the bank value decreases. A disclosure lag increases the occurrence of bank runs when bank assets have high growth.

Assets with high growth rates decrease the manager's risk appetite which reduces the probability of a bank run. Keeping the value of the assets constant, the bank manager selects low risk projects when the project has relatively higher growth opportunities. This occurs because a high growth rate increases the manager's payoff and hence she relies less on risk-shifting to increase her payoff. With low risk and high growth assets, creditors' aggregate response is to run less frequently. Hence, as growth opportunities increase we see banks that have lower probability of default.

This paper is part of the disclosure in the financial industry literature. Gigler, Kanodia, Sapra, Venugopalan (2013) (hereafter GKS_V) study the frequency of disclosure that should be required for public firms. They show that when the bank manager can endogenously take decisions, frequent disclosure does not necessarily imply economic efficiency. In a similar flavor, our model says that a shorter disclosure lag does not necessarily improve bank value as the manager might risk-shift. In a model with multiple frictions and when the bank's decision is endogenous, price efficiency does not imply economic efficiency. Morris and Shin (2002) show the trade-off between market discipline and strategic concerns that cause coordination problems. We provide a bank specific structure and model explicitly the information disclosure lag, which allows us to identify when lags are beneficial by looking at the bank assets.

The situation of study presented in this paper applies to disclosure policy for financial institutions where a lag is considered or implemented. As motivated in this introduction, it could be applied to the analysis of disclosure of participants, volumes and collateral at the discount window of the Federal Reserve. Before the Dodd-Frank act, identities of borrowers at the discount window

where kept confidential. The Dodd-Frank act mandated disclosure with a two year lag. Our model applies to this situation since the trade-offs between incentive provision and bank runs has been recognized as important in the discount window disclosure debate⁴. Another situation where the model presented is relevant is regarding the disclosure of banks' stress tests. Disclosure of stress tests has been controversial for several reasons, including the possibility of bank runs. This has been documented both in academic papers (Goldstein and Sapra (2014)) and in the press⁵. A lag in disclosure is a tool that bank regulators might consider in order to balance the benefits of market discipline and costs of bank runs.

There are other contexts for which the framework presented in this paper might be relevant. In accounting, there is interest in understanding the impact of accounting rules based on mark-to-market and historic cost. A reinterpretation of my model allows for short lags to be seen as mark-to-market accounting while longer lags emulate historic cost. The model could shed light into understanding the impact of accounting rules when taking into consideration the distribution of assets held by banks, the coordination problem induced by short-term debt and managerial risk-shift. Finally, central banks have been using disclosure lags in a variety of situations. In many situations it has slowly shifted from a no disclosure policy to a disclosure with lag policy. For example, the decisions of the Federal Open Market Committee (FOMC) were not announced twenty years ago. The Fed gradually increased the degree of communication and currently it releases its interest rate decision immediately after the meeting and the minutes with a lag of three weeks⁶.

2 Model

The model builds on He and Xiong (2012). The model is written in continuous time with an infinite horizon. The bank invests in a long-term asset by rolling over short-term debt financed by

⁴See "Banks Face Borrowing Stigma" <http://on.wsj.com/1L5CXb6>

⁵See "Lenders Stress Over Test Results" <http://on.wsj.com/1PWhsOh>

⁶See "A Short History of FOMC Communication" <http://www.dallasfed.org/research/ecllett/2013/el1308.cfm>

a continuum of small creditors.

2.1 Assets, the banker and moral hazard

The bank's asset holding is normalized to one unit. The bank borrows \$1 at time $t = 0$ to buy the asset. Once the asset is in place, it generates a constant stream of cash flow $r dt$ over the interval $[t, t + dt]$. At a random time τ_ϕ , which arrives according to a Poisson process with intensity $\phi > 0$, the asset matures with a final payoff of y_{τ_ϕ} . The advantage of assuming a random asset maturity is that it makes the expected life of the project constant and equal to $1/\phi$.

The final payoff of the asset evolves according to a geometric Brownian motion with drift μ and volatility σ ,

$$\frac{dy_t}{y_t} = \mu dt + \sigma dW_t, \quad (1)$$

y_0 given and $W(t)$ is a standard Brownian motion, μ is the growth rate of the final payoff and σ is the instantaneous volatility.

The bank's asset generates the constant cash flow $r dt$ and the random final payoff y_{τ_ϕ} . The value of the project is the expected discounted future cash flows

$$F(y_t) = E_t \left[\int_t^{\tau_\phi} e^{-\rho(s-t)} r ds + e^{-\rho(\tau_\phi-t)} y_{\tau_\phi} \right] = \frac{r}{\rho + \phi} + \frac{\phi}{\rho + \phi - \mu} y_t, \quad (2)$$

where $\frac{r}{\rho + \phi}$ is the present value of the constant cash flow and $\frac{\phi}{\rho + \phi - \mu} y_t$ is the present value of the final payoff. Since the fundamental value of the bank is a linear function of y_t , we refer to y_t as the bank fundamental.

A. Risk-shifting

The bank manager controls the risk of the assets, in particular, she chooses the asset's final-payoff volatility (σ). The bank manager chooses at time 0 a risk level $\sigma \in [\sigma_L, \sigma_H]$. Equivalently, the banker chooses a combination of a project with low risk and a project with high risk. The value

of σ is observable, but assumed not contractible. Risk-shifting is a feature not present in He and Xiong (2012). A different version of risk-shifting is studied by Cheng and Milbradt (2011).

2.2 Debt financing, runs and liquidation

The bank financing, runs and liquidation follow closely He and Xiong (2012). I present a brief review for completeness. The bank finances the asset by issuing short-term debt. That is, each debt contract lasts for an exponentially distributed amount of time with mean $1/\delta$. Once an individual contract expires, the creditor chooses whether to roll over the debt or run. The maturity shocks are independent across creditors so that each creditor expects some other creditors' contracts to mature before his.

In aggregate, a fraction δdt of the banks debt matures over the time interval $[t, t + dt]$. When maturing creditors choose to run, the bank must find financing from other sources or it will be forced into bankruptcy. I assume that the bank has access to a credit line that supplies the financing required. When a run occurs, there is a probability $\theta \delta dt$ that the credit line will fail to provide the required financing. The parameter $\theta > 0$ measures the reliability of the credit line. A low value of θ means that the credit line will sustain the bank run with high probability. If the credit line fails, the bank is forced to liquidate assets in an illiquid secondary market. I assume that the asset's liquidation value is a fraction $0 < \alpha < 1$ of the fundamental value of the project

$$\begin{aligned}
 \mathcal{L}(y_t) &= \alpha F(y_t) \\
 &= \frac{\alpha r}{\rho + \phi} + \frac{\alpha \phi}{\rho + \phi - \mu} y_t \\
 &= L + l y_t,
 \end{aligned} \tag{3}$$

where $L = \frac{\alpha r}{\rho + \phi}$ and $l = \frac{\alpha \phi}{\rho + \phi - \mu}$.

2.3 Information sets

The model departs from He and Xiong (2012) in the setup of the information structure. A regulator has control over the disclosure of the bank's fundamental, y_t . The regulator discloses the value of the bank fundamental with a lag $I \geq 0$. This means that at time $t > 0$ creditors learn what was the bank's fundamental value at time $t - I$, y_{t-I} . Creditors are aware of the fact that they are receiving outdated information and will form rational expectations about the bank's fundamental value today. Mathematically, creditors observe x_t , the bank fundamental process with lag, where

$$x_t = y_{t-I} \quad (4)$$

with $I \geq 0$. Knowing that y_t evolves according to equation (1), creditors anticipate that x_t evolves according to a geometric Brownian motion with the same drift and variance as in equation (1)

$$\frac{dx_t}{x_t} = \mu dt + \sigma dW_t, \quad x_I = y_0. \quad (5)$$

I assume that the bank manager is an insider and observes the bank fundamental, y_t , with no lag. Moreover, she is aware of the fact that creditors observe a lagged fundamental value and she will use this information in making optimal choices.

Figure 2 presents a timeline describing the information structure of the model. The lag in disclosure is set at a value $I \geq 0$, and is common knowledge to all parties. The solid line represents a realization of the fundamental process, $\{y_u\}_{u \leq t}$, which is observed by the manager. The dashed line represents the corresponding lagged process $\{x_u\}_{u \leq t}$, which is the information available to the creditors at time t . At time 0, the bank manager chooses the riskiness of the project, $\sigma \in [\sigma_L, \sigma_H]$, based on the current value of the fundamental, y_0 . The value of σ is fixed after $t \geq 0$.

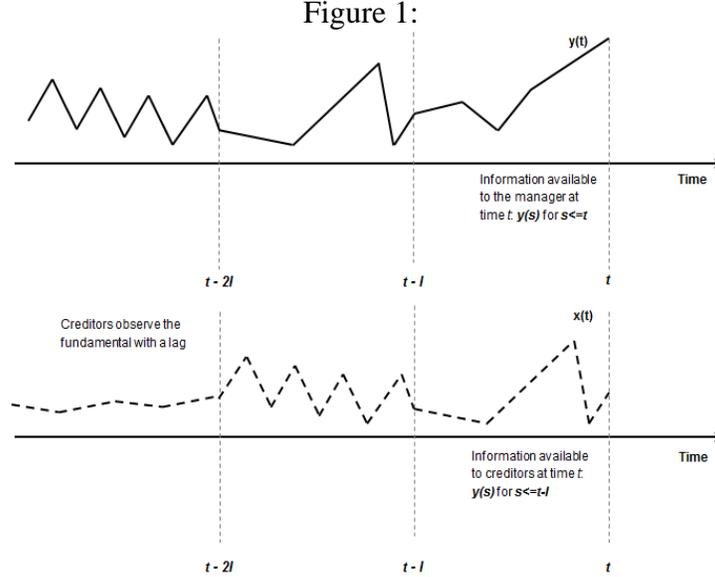


Figure 2: Information Structure for a Disclosure Lag $I > 0$. The solid line represents the fundamental process, $y(t)$, observed by the manager. The dashed line represents the lagged process $x(t)$, which is the information available to the creditors at time t .

3 Creditor's and manager's problem

3.1 An individual creditor's problem

I follow He and Xiong (2012) and analyze the rollover decision for an individual creditor by taking as given that other creditors use a monotone strategy. A monotone strategy indicates that all creditors whose debt is maturing decide to rollover when the bank's fundamental x_t is greater than a threshold y_* . When $x_t \leq y_*$ creditors whose debt matures decide to run.

There are two possible outcomes for the bank. Either the project's final payoff is realized or the bank is prematurely liquidated after a run. These events are not controlled directly by an individual creditor. However, once an individual's debt matures, he can decide whether to rollover the debt or not. Each creditor receives interest payments at a rate r per unit of time until

$$\tau = \min(\tau_\phi, \tau_{\theta\delta}, \tau_\delta),$$

which is the earliest of the following three events: Project completion, forced liquidation after a

run and debt expiration without rollover, respectively. When debt matures, creditors receive the face value of the debt back and have the option to rollover their position by buying the new debt.

With a risk neutral creditor, the value of one unit of debt is given by the value function

$$\begin{aligned}
V(x_t) = & E_t \left[\int_t^\tau e^{-\rho(s-t)} r ds + e^{-\rho(\tau-t)} \{ \min(1, y_\tau) \mathbf{1}_{\{\tau=\tau_\phi\}} \right. \\
& + \min(1, \mathcal{L}(y_\tau)) \mathbf{1}_{\{\tau=\tau_{\theta\delta}\}} \\
& \left. + \max_{\text{run or rollover}} (1, V(x_\tau; y_*)) \mathbf{1}_{\{\tau=\tau_\delta\}} \right], \tag{6}
\end{aligned}$$

where $\mathbf{1}_{\{\cdot\}}$ takes the value 1 when the statement in brackets is true and 0 otherwise. The value for an individual creditor has four components, represented by the four terms in the right hand side of equation (6). First, he will receive coupon payments at rate r while the bank is alive. Second, when the asset matures, the creditor will be paid in full if the final payoff is greater than 1, and y_t if it is lower than 1, $\min(1, y_\tau)$. Third, if the bank is forced into bankruptcy, the creditor will be paid in full only when the asset value after liquidation costs is greater than 1, otherwise he will receive the asset which can be liquidated for $\mathcal{L}(y_\tau)$, $\min(1, \mathcal{L}(y_\tau))$. The last term represents the option that each creditor has to decide whether to rollover or run when the debt matures. The expectation in (6) is conditional on the information available to the creditor at time t , that is, the lagged fundamental value process x_t .

The Appendix derives the HJB equation for the value function $V(x_t)$,

$$\begin{aligned}
\rho V(x_t; y_*) = & \mu x_t V_x + \frac{\sigma^2}{2} x_t^2 V_{xx} + r + \phi [E_t [\min(1, y_t)] - V(x_t; y_*)] \\
& + \theta \delta \mathbf{1}_{\{x_t < y_*\}} [E_t [\min(1, \mathcal{L}(y_t))] - V(x_t; y_*)] \\
& + \delta \max_{\text{run or rollover}} (1 - V(x_t; y_*), 0)
\end{aligned} \tag{7}$$

where $E_t[\cdot]$ is the expectation taken with the information available to the creditor at time t . Even though the creditor knows that the current time is t , he needs to estimate the current value of y_t from the information provided by the lagged process, $x_t = y_{t-1}$. Hence, a creditor's estimate of the

payoff in case of project completion is $E_t [\min(1, y_t)] = E_t [\min(1, x_{t+I})]$. Similarly, the creditor's estimate of the payoff in case of bankruptcy at time t is $E_t [\min(1, \mathcal{L}(y_t))] = E_t [\min(1, \mathcal{L}(x_{t+I}))]$. The information structure of the model leads to a generalized version of the HJB equation in He and Xiong (2012). Of course, when $I = 0$, we recover the value function in He and Xiong (2012).

The left hand side of equation (7), $\rho V(x_t; y_*)$, represents the creditor's required return. The right hand side represents the expected increments on the continuation value. The first two terms, $\mu x_t V_x + \frac{\sigma^2}{2} x_t^2 V_{xx}$, capture the change in continuation value caused by the fluctuation in the bank fundamental. The next four terms represent the components that appeared in the integral form of the value function, equation (6). That is, the coupon payment rate, r , the value change when the project matures at time τ_ϕ ($\phi [E_t [\min(1, y_t)] - V(x_t; y_*)]$), the value change caused by a forced liquidation, $\theta \delta \mathbf{1}_{\{x_t < y_*\}} [E_t [\min(1, \mathcal{L}(y_t))] - V(x_t; y_*)]$, and the option to run when the debt contract expires, $\delta \max_{\text{run or rollover}} (1 - V(x_t; y_*), 0)$, respectively.

A creditor whose debt matures will choose to rollover whenever the value of doing so is higher than the debt's face value of 1. In other words, if the value function only crosses 1 at the point x' ($V(x'; y_*) = 1$), then x' is the optimal cutoff for the creditor. Whenever debt matures and $x_t < x'$, the creditor will not rollover the debt. On the other hand, when $x_t \geq x'$, the creditor finances a new debt contract.

I solve for symmetric monotone equilibria for which the threshold used by creditors is the same, y_* . Therefore, a condition to determine the threshold y_* is that $V(y_*; y_*) = 1$.

3.2 The bank manager's problem

The bank manager holds the firm's equity. Given a rollover threshold, y_* , the manager maximizes the total value of the residual claim by choosing the project riskiness, σ . The manager's choice is made at time 0 and is held constant afterwards. The value of equity at time t is

$$Q(y_t; y_*, \sigma) = E_t [e^{-\rho(\tau-t)} \{ \max(y_\tau - 1, 0) \mathbf{1}_{\{\tau=\tau_\phi\}} + \max(\mathcal{L}(y_\tau) - 1, 0) \mathbf{1}_{\{\tau=\tau_{\theta\delta}\}} \}], \quad (8)$$

where $E_t[\cdot]$ is the expectation conditional on the information available to the manager at time t , the non-delayed fundamental value y_t . The value function of the manager has two components, represented by the two terms on the right hand side of (8). The first one represents the equity payoff when the project ends, $(y_\tau - 1)^+$, and the second one represents the payoff after a forced liquidation, $(\mathcal{L}(y_\tau) - 1)^+$.

Figure 4 presents a timeline describing the information available to the manager at time t . As an insider of the bank, I assume that the manager can observe the fundamental value with no delay, $\{y_s\}_{s \leq t}$. This is shown in Figure 4 as the black solid-line. The manager is also aware that creditors are making optimal choices based on lagged information. This means that at time t , the manager can anticipate how creditors will behave, rollover the debt or not, from t until $t + I$. It also means that the value function depends on the path of the fundamental process from $t - I$ to t , $\{y_s\}_{t-I \leq s \leq t}$.

The manager's value function is the sum of two terms: The value of events that happen from t until $t + I$ and events that happen from $t + I$ onwards:

$$Q(\{y_s\}_{t-I \leq s \leq t}; y_*, \sigma) = Q^1(\{y_s\}_{t-I \leq s \leq t}; y_*, \sigma) + e^{-\rho I} P(\tau \geq t + I) Q^2(y_t; y_*, \sigma) \quad (9)$$

where $Q^1(\{y_s\}_{t-I \leq s \leq t}; y_*, \sigma)$ represents the part of the value function that is path dependent and $Q^2(y_t; y_*, \sigma)$ represents the value from $t + I$ onwards which is not path dependent.

I study the steady state situation in which the value function for the manager is path independent. Specifically, I study the behavior of $Q^2(y_{t-I}; y_*, \sigma)$. This modelling choice allows me to focus on how the manager and creditors change behavior as the delay in information changes while keeping the path dependence complications manageable. Moreover, this scenario is consistent with a case in which there has been a lag in disclosure and an increase to more disclosure is being considered.

The value for the manager at time t from events that occur after $t + I$ is

$$Q^2(y_t; y_*, \sigma) = E[e^{-\rho(\tau-t)} \{(y_\tau - 1)^+ \mathbf{1}_{\{\tau=\tau_\phi\}} + (\mathcal{L}(y_\tau) - 1)^+\} \mathbf{1}_{\{\tau=\tau_{\theta\delta}\}} | A_2], \quad (10)$$

Figure 3:

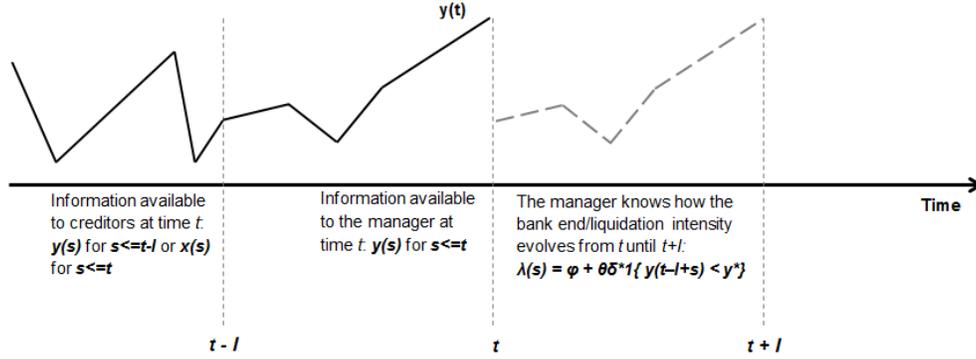


Figure 4: Manager's Information. The manager observes the fundamental value with no delay, $y(t)$, black solid-line. The manager is aware that creditors decide based on lagged information, dashed grey line

with $A_2 = \{y_t, \tau \geq t + I\}$. The set A_2 represents the information for events that occur after $t + I$.

The Appendix presents the derivation of the HJB equation for the equity value conditional on the bank surviving until time $t + I$, $Q^2(y_t; y_*, \sigma)$,

$$\begin{aligned} \rho Q^2(y_t; y_*, \sigma) &= \mu y_t Q_y^2 + \frac{\sigma^2}{2} y_t^2 Q_{yy}^2 + \phi [E_t[(y_{t+I} - 1)^+] - Q^2] \\ &+ \theta \delta \mathbf{1}_{\{y_t < y_*\}} [E_t[(\mathcal{L}(y_{t+I}) - 1)^+] - Q^2]. \end{aligned} \quad (11)$$

The left hand side of equation (11), $\rho Q^2(y_t; y_*, \sigma)$, represents the banker's required return. The right hand side represents the expected increments on the continuation value. The first two terms, $\mu y_t Q_y^2 + \frac{\sigma^2}{2} y_t^2 Q_{yy}^2$, capture the change in continuation value caused by the fluctuation in the bank fundamental. The next two terms capture the final value for the banker. The third term, $\phi [E_t[(y_{t+I} - 1)^+] - Q^2]$, captures the expected continuation value change when the project matures at time τ_ϕ . The last term, $\theta \delta \mathbf{1}_{\{y_t < y_*\}} [E_t[(\mathcal{L}(y_{t+I}) - 1)^+] - Q^2]$, represents the expected continuation value change caused by a forced liquidation.

3.2.1 Optimization problem

Given an initial value of the process, y_{t-I} , and an optimal creditor's cutoff, y_* , the manager chooses the project riskiness that maximizes the value of equity at time 0,

$$\sigma_* = \arg \max_{\sigma_L \leq \sigma \leq \sigma_H} Q^2(y_0; y_*, \sigma), \quad (12)$$

subject to

$$\frac{dy_t}{y_t} = \mu dt + \sigma dW_t, \quad y_0 \text{ given}, \quad (13)$$

and

$$\begin{aligned} \rho Q^2(y_t; y_*, \sigma) &= \mu y_t Q_y^2 + \frac{\sigma^2}{2} y_t^2 Q_{yy}^2 + \phi [E_t[(y_{t+I} - 1)^+] - Q^2] \\ &\quad + \theta \delta \mathbf{1}_{\{y_t < y_*\}} [E_t[(\mathcal{L}(y_{t+I}) - 1)^+] - Q^2]. \end{aligned} \quad (14)$$

4 Bank Runs and Asset-Risk Equilibrium

Given a project riskiness, σ , we limit attention to monotone equilibria in which each creditor's rollover strategy is monotone with respect to the bank fundamental observed at time t , x_t . This means that we are interested in deriving a cutoff, y_* , such that whenever $x_t < y_*$, the optimal decision for maturing creditors is to run on the bank and when $x_t \geq y_*$, the optimal strategy is to rollover. Moreover, we explore symmetric monotone equilibria in which each creditor's optimal choice, x' , must be equal to the other creditors' threshold y_* . Therefore, a condition for determining the equilibrium threshold is $V(y_*; y_*) = 1$.

The analysis requires some parameter restrictions in order to be meaningful. We work with the same parameter restrictions as in He and Xiong (2012).

Definition 1 *A pair (σ_*, y_*) , project volatility and run threshold, is a Nash equilibrium if $V(y_*; y_*) = 1$ and $\sigma_* = \arg \max_{\sigma_L \leq \sigma \leq \sigma_H} Q^2(y_{t-I}; y_*, \sigma)$. That is, the threshold is optimal for a creditor that takes σ_* as given and the manager maximizes the equity value (10) taking the optimal cutoff y_* as*

given.

4.1 Equilibrium Analysis

We perform a numerical analysis that will serve to understand the basic properties of the model.

A. Parameter values

Table 1 presents the baseline parameter values. Except for the volatility (which is endogenously chosen by the manager), we use the same parameters values as in He and Xiong (2012).

Parameter	Value	Interpretation
ρ	1.5%	Discount rate
r	7%	Cash flow rate from project
ϕ	0.077	Intensity of terminal value realization
α	55%	Liquidation discount
μ	1.5%	Drift of asset final payoff
δ	10	Intensity at which debt matures
θ	5	Intensity of credit line failure
y_0	1.4	Fundamental value at time t
σ_L	2%	Low-risk volatility
σ_H	12%	High-risk volatility

Table 1. Baseline parameters.

4.2 Benchmark: No runs and no risk shifting

It is informative to conduct an analysis of the model when there are no runs or risk shifting frictions. If there are no runs but a single creditor that holds a perpetual bond, then the manager faces no trade off when choosing volatility. The bank manager benefits with risk and therefore she chooses the maximum risk possible, σ_H . If at the riskiness level σ_H the creditors value is

greater than the initial investment required of \$1, then financing will occur. Otherwise, there is no financing. The firm value is independent of σ , but a transfer of value occurs from the creditor to the manager as the manager chooses more risky projects.

If the risk of the project is fixed, we are back at the model by He and Xiong (2012). Runs are costly because of illiquid asset values. Lowering the run threshold increases bank value.

4.3 Equilibrium with no disclosure lag

Consider a situation in which there is no lag in disclosure, or $I = 0$. Figure 6 plots the manager's and creditors' optimal behavior. The solid line plots the optimal volatility, σ_* , as a function of the cutoff y_* . The higher the run cutoff y_* , the lower the optimal volatility σ_* . For low values of y_* , the probability that the fundamental reaches y_* is low. Since the manager is long a call option on the fundamental of the bank, he chooses the maximum volatility possible. As the run cutoff increases, the marginal value of volatility for the manager decreases because it is more likely for a run to occur.

The dash line in Figure 6 shows the optimal threshold, y_* , as a function of the fundamental's volatility, σ . The running cutoff is an increasing function of asset risk. The creditor's final payoff, $E_t[\min(1, y_t)]$, is equal to the payoff obtained by a short position on a put option and long position on a bond, that is, $E_t[\min(1, y_t)] = E_t[1 - \max(1 - y_t, 0)]$. A higher volatility makes the short position on the put less valuable, harming creditors, which in aggregate choose a higher running threshold.

A pair (σ_*, y_*) is an equilibrium if σ_* is consistent with y_* . This means that the intersection of the solid line and dash line determines an equilibrium in Figure 6. The equilibrium is given by the pair $(\sigma_*, y_*) = (3.24\%, 0.9)$. This is an equilibrium where there is some risk-shifting and we will observe a run whenever the fundamental value x_t gets below 0.9.

Figure 5:

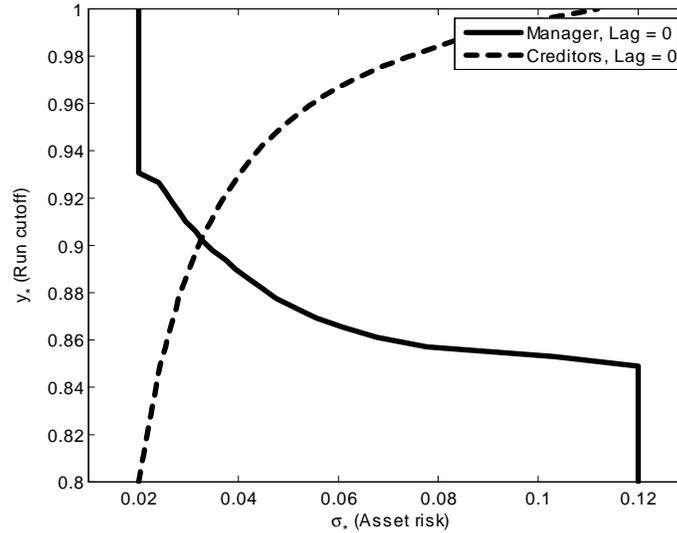


Figure 6: Best Responses and Equilibrium with no Lag. The solid line plots the optimal volatility as a function of the cutoff y^* . The dashed line plots the optimal threshold, y^* as a function of the fundamental's volatility. The equilibrium occurs when risk is consistent with y^* .

4.4 Disclosure Lag Equilibrium

Now we turn our attention to the equilibrium analysis when there is a disclosure lag, $I > 0$. Figure 8 plots the banker's and creditors' optimal strategy using the baseline parameters for two lag values, $I \in \{0, 2.5\}$. The black-solid line plots the bank manager's optimal volatility (σ_*) as a function of the cutoff y_* for $I = 0$. The black-dash line plots the creditors' optimal run threshold, y_* , as a function of the volatility σ for $I = 0$ (Same lines as in Figure 6). The equilibrium is given by the intersection of the lines at the point $(\sigma_*, y_*) = (3.24\%, 0.9)$.

Figure 8 also plots the optimal choice taken by the bank creditors and manager when there is a disclosure lag of $I = 2.5$ years. The grey-solid line plots the optimal risk level chosen by the bank manager (σ_*) as a function of the cutoff y_* for $I = 2.5$. The grey-dash line plots the optimal run threshold chosen by creditors, y_* , as a function of the volatility σ for $I = 2.5$. The intersection of these lines determines the equilibrium, which is given by the pair $(\sigma_*, y_*) = (3.84\%, 0.89)$. When compared to the zero lag case, this equilibrium has a lower roll-over threshold and higher asset risk.

By studying equation (7), we can understand why the rollover threshold (y_*) decreases with an increase in the disclosure lag. The expected change in continuation value when the project ends is given by the term $\phi [E_t [\min(1, y_t)] - V(x_t; y_*)]$. As noted previously,

$$E [\min(1, y_t)] = E [1 - \max(1 - y_t, 0)],$$

so that the creditors final payoff is equal to having a long position in a bond and a short position in a put option written on the final payoff (y_t). The derivative of the put option value with respect to the lag I is

$$\frac{\partial E_t [\max(1 - y_t, 0)]}{\partial I} = N'(-d_-) \frac{\sigma}{2\sqrt{I}} - x_t \mu e^{\mu I} N(-d_+), \quad (15)$$

where

$$d_{\pm} = \frac{1}{\sigma\sqrt{I}} \left(\ln[x_t] + \left(\mu \pm \frac{1}{2}\sigma^2 \right) I \right) \quad (16)$$

and $N(\cdot)$ is the cumulative distribution of the standard normal distribution and $'$ indicates the first derivative.

When the volatility of the project σ is low relative to the drift μ , equation (15) indicates that the put option value decreases as the lag I increases. In other words, creditors, who are short a put option, see an increase in value as the lag I increases. Intuitively, with a positive drift, a longer lag makes it more likely for the final asset payoff to end above the face value of the debt. In this situation, bad news that occurred in the past are smoothed out thanks to the expectation that the process recovers on average.

If the volatility of the project σ is high relative to the drift μ , an opposite effect takes place. Since the put option increases in value with the delay of information, creditors are worse off and hence choose to run sooner. In aggregate they have a higher threshold. Figure 8 illustrates that this is the case for the higher volatilities in the allowed range.

A lag in disclosure also affects the optimal behavior for the manager. A longer lag in disclosure makes risk-shifting more attractive because the marginal value of volatility increases. The expected continuation value change when the project ends is given by the term $\phi [E_{t-I} [(y_t - 1)^+] - Q^2(y_t; y_*)]$

which indicates that the manager is long a call option on the final payoff of the asset for which maturity is equal to I . The derivative of the call option value with respect to the lag I is given by

$$\frac{\partial E_{t-I} [\max(y_t - 1, 0)]}{\partial I} = N'(d_-) \frac{\sigma}{2\sqrt{I}} + y_t \mu e^{\mu I} N(d_+), \quad (17)$$

where d_{\pm} are given by equation (16). The derivative of the call option is always positive.

It is not clear whether the bank value is higher or lower with a disclosure lag. Holding everything else constant, a higher value for σ_* decreases the value of the bank. This occurs because a higher volatility rate increases the likelihood of the fundamental value crossing the roll over threshold y_* , which will cause creditors to run and costly liquidate the bank.

On the other hand, a lower run cutoff y_* increases the bank value. This occurs because a rollover freeze is less likely and hence inefficient liquidations occur less frequently, which is value increasing.

Figure 8 shows that as the lag in disclosure increases, the equilibrium volatility σ_* increases and the cutoff y_* decreases, making unclear whether the bank value increases or decreases. Both extremes, no disclosure or immediate disclosure, present some cost to the bank and hence there is room to propose the lag in disclosure as a policy tool. Which effect dominates, the reduction in the cutoff y_* the or increase in volatility σ_* , will depend on the slopes of the curves.

4.5 Disclosure lags and the probability of a run

A disclosure lag causes the bank value to change. As the lag in disclosure changes, the equilibrium run-cutoff and the equilibrium project riskiness change affecting the probability of a run. A higher probability of a run decreases bank value because asset liquidation is costly. Given (σ_*, y_*) , the probability of a run can be computed by using

$$P_{y_0} \left(\inf_{0 \leq s \leq \tau_\delta} y_s \leq y_* \right) = \left(\frac{y_*}{y_0} \right)^{\sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\phi}{\sigma^2}} + \left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)}. \quad (18)$$

Figure 7:

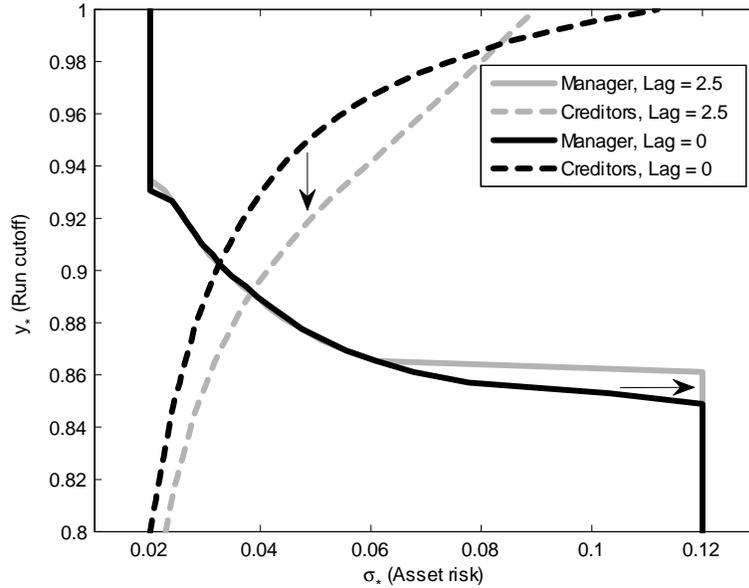


Figure 8: Equilibrium with Disclosure Lag for $I=2.5$ years. The solid line plots the optimal volatility as a function of the cutoff y^* . The dashed line plots the optimal threshold, y^* , as a function of the fundamental's volatility. The equilibrium occurs when risk is consistent with y^* .

Equation (18) is increasing in y_* and it is also increasing in σ^2 provided that $\mu \geq \sigma^2/2$. That is, higher run cutoffs and riskier projects increase the probability of a run.

A lower probability of a run increases the bank value. For the disclosure lag to increase the value of the bank, the reduction of the run cutoff should compensate the increase in project risk. Alternatively, the reduction in risk should be enough to compensate for an increase in the cutoff.

Figure 9 adds contour plots to the equilibrium analysis with two lags of Figure 8. As expected from equation (18), the probability of a run increases with the riskiness of the project and with the run cutoff. Both equilibriums, the equilibrium with and with no disclosure lag are located in a zone where the probability of a run is very low. Since the run probability stays almost equal in the presence of a lag, the bank value does not change.

Figure 11 plots the bank value, run probability and expected bailout costs as a function of the disclosure lag. As expected, the bank value is flat for low values of the lag. For a disclosure lag of three or more years, the bank value decreases because the increase in risk is not compensated by

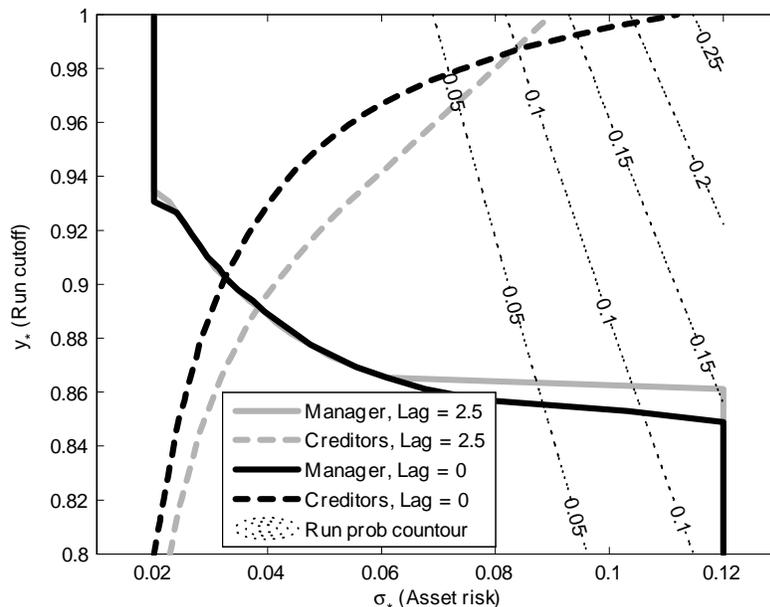


Figure 9: Probability of a Bank Run. The dotted lines present contour plots for the probability of a run. Both equilibria, with a lag of 2.5 years and no lag, occur in a low probability area.

the change in the run cutoff.

5 Optimal Disclosure Lag

The parameters from He and Xiong (2012) deliver an equilibrium that has a very low probability of default. According to Figure 11 it increases from 0 up to 0.2% when the disclosure lag increases from 0 to 6 years. In essence, the equilibrium with no information delay has no real probability of default, and hence the changes in bank value are small.

We modify the parameters to study how the bank value changes in response to a lag in information disclosure when the probability of a run is significant. Keeping the value of the asset constant, we let μ decrease so that $\mu = 0.5\%$ and $r = 8.42\%$. A lower drift (μ) has effects on both the choices of the bank manager and creditors. The bank manager selects a higher volatility when the drift (μ) is lower, keeping everything else fixed. The reason is that a lower drift decreases the final asset payoff and hence the manager optimally increases her risk bet. For creditors, a lower drift

Figure 10:

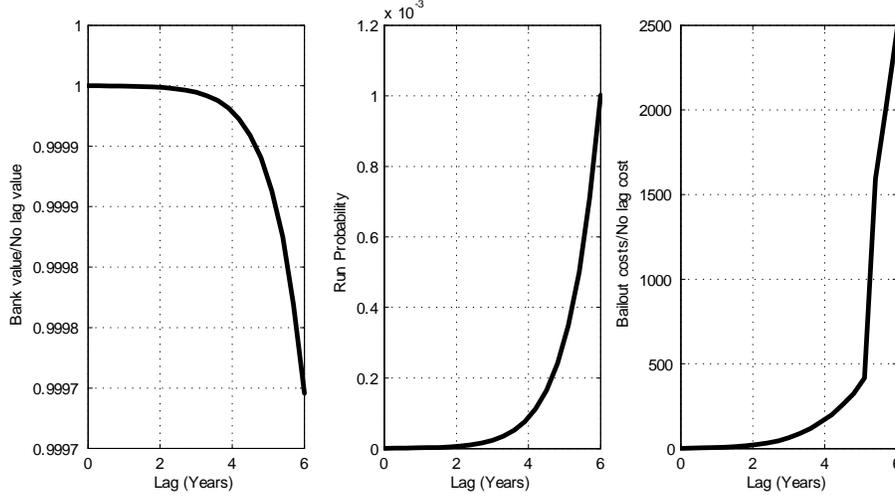


Figure 11: Bank Value, Run Probability and Bailout Costs vs Disclosure Lag. Bank assets have a high-growth rate.

(μ) in the final payoff increases their incentives to run as they see lower quality in the assets.

Increasing the coupon rate (r) affects the optimal response by creditors. With a higher coupon rate, creditors incentives to run decreases they see a better prospect for the asset of the bank. The bank manager's behavior is mildly affected by a change in the coupon rate.

A lower drift (μ) of the final payoff and a higher coupon rate (r) make the equilibrium to shift to higher volatilities, which allows for more action in the bank value as the disclosure lag is introduced.

Figure 12 plots the bank value, run probability and bailout costs as the disclosure lag changes from 0 to 6 years. As the delay increases from 0 to 2 years, the probability of a run decreases and the bank value increases. The bank value attains a maximum and then starts decreasing for values higher than 3 years. The probability of a run increases from 3 years onwards. The probability of a run decreases because the equilibrium risk reduction effect dominates the increase in the run cutoff. After 2 years, the increase in the run cutoff dominates and hence the probability of a run increases, which means that bank value decreases.

There are three forces that cause the bank value to change when the lag changes. First, the

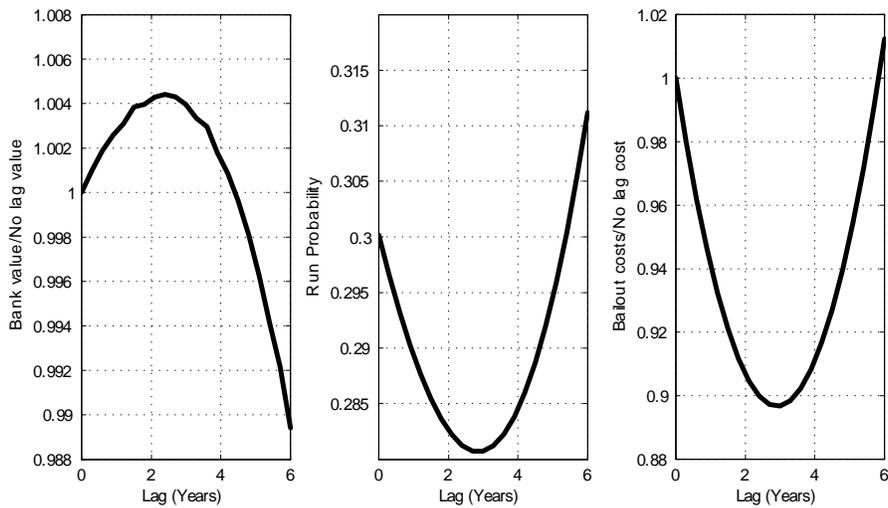


Figure 12: Bank Value, Run Probability and Bailout Costs vs Disclosure Lag. Bank assets have low-growth rates. An optimal disclosure lag occurs between 2 and 3 years.

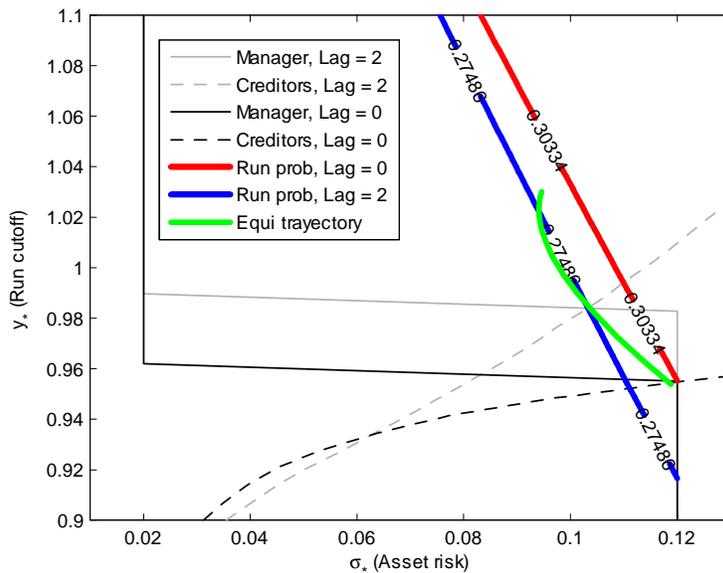


Figure 13: Run probability and Disclosure Lag with Low Growth Assets

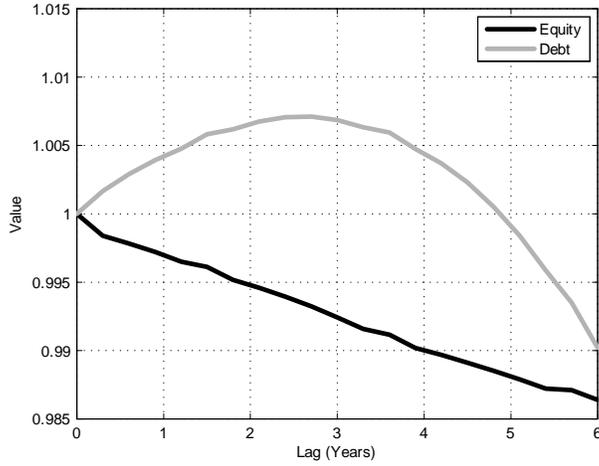


Figure 14: Bank's Equity and Debt vs Disclosure Lag.

optimal roll over cutoff for creditors changes. In particular, a decrease in the cutoff (everything else equal), increases both the creditors and manager's position and hence it increases the bank value. The second force at play is the optimal volatility selected by the manager. An increase in optimal volatility decreases bank value because it makes more likely for the fundamental to cross the roll over threshold. The third force is the extra volatility that one gets from the delay. Today's estimate of the fundamental value y_t , $E_t[y_t] = x_t e^{\mu I}$ is kept constant by setting, $x_t = 1.4e^{-\mu I}$. However, as the lag increases, the increase in volatility decreases the bank value,

$$\begin{aligned} \rho A(x_t; y_*, \sigma) &= \mu x_t A_x + \frac{\sigma^2}{2} x_t^2 A_{xx} + r + \phi [x_t e^{\mu I} - A] \\ &\quad + \theta \delta \mathbf{1}_{\{x_t < y_*\}} [L + l x_t e^{\mu I} - A]. \end{aligned} \quad (19)$$

6 Optimal delay and bank characteristics

Using the parameters in He and Xiong (2012), we have seen that the run probability increases as the disclosure lag increases. This translates into a lower bank value for higher lags. The reason for this result is that the decrease in the run cutoff effect is dominated by the increase in the risk of the project.

Figure 15:

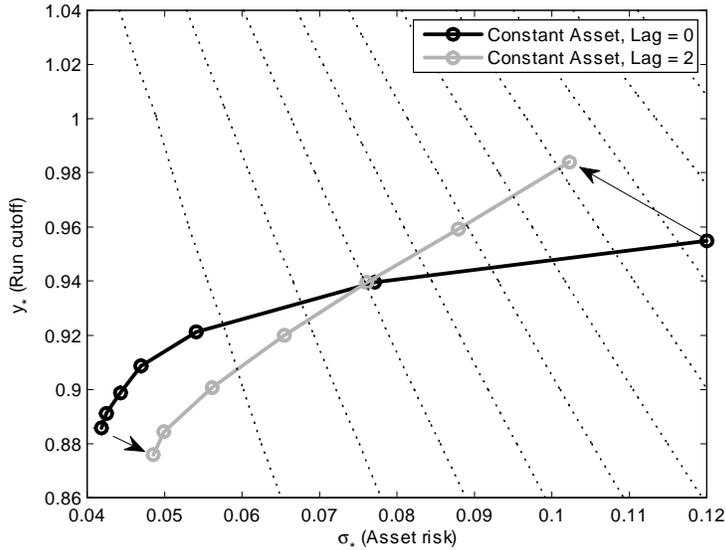


Figure 16: Equilibrium with Constant Asset Values and Disclosure Lag. High-growth assets are in the bottom left of the graph and low-growth assets are in the top right.

In the previous section, we decreased μ and increased r while keeping constant the initial value of the assets. An increase in r decreases the run cutoff keeping other parameters constant. The result is the equilibrium occurs at higher values of σ where the probability of a run is higher. As the disclosure lag increases, the equilibrium has lower asset risk (σ_*) and higher run cutoff (y_*). The effect of lower asset risk dominates, decreasing the run probability and increasing the bank value. The delay of information is beneficial. Moreover, there is an optimal delay that maximizes the bank value.

Increasing the coupon rate r and decreasing the growth rate μ (Keeping the firm value constant) is shifting value from the final payoff to the continuously paid coupons. Another interpretation is that increasing r and decreasing μ is decreasing the average cash-flows maturity of the asset. Yet another interpretation is that the shift represents a move to an asset where a greater proportion comes from the fixed income payments relative to random final payoffs.

Figure 16 plots how the equilibrium changes when there is an increase in r and a decrease in μ while keeping the asset value constant. The black line with circles plots how the equilibrium

changes from a high drift (μ) and low coupon rate (r) (Benchmark parameters, bottom left) to a low drift and high coupon rate. The equilibrium moves from a low asset risk with low run threshold ($\sigma = 4\%$, $y_* = 0.88$) to a higher asset risk and higher run threshold ($\sigma = 12\%$, $y_* = 0.95$).

The black line in Figure 16 plots how the equilibrium changes when assets become more liquid (high r and low μ). The equilibrium riskiness of the assets, σ_* , increases because the manager compensates the low growth opportunities by increasing the bet on volatility. As the asset becomes more risky, creditors' optimal response is to increase the run threshold (y_*). A project with low growth opportunities generates high probabilities of a run and lower bank values.

We can state the previous result in a different way. Relatively large duration assets decrease the manager's risk appetite and this reduces the bank run probability. Keeping the value of the assets constant, the bank manager selects low risk projects when the project has relatively higher growth opportunities. This occurs because a high growth rate increases the manager's payoff and hence she depends less on risk-shifting to increase her utility. With low risk, creditors' aggregate response is to run less frequently. Hence, as growth opportunities increase we see banks that have lower probability of default.

The grey line with circles in Figure 16 plots how the equilibrium changes when there is an increase in r and a decrease in μ while keeping the asset value constant for a disclosure lag of 2 years. In the bottom left corner, when the asset has relative low coupon rate and high drift, we are back the situation presented in Figure 8. Increasing the lag produces a more risky project and lower run cutoff. The probability of a run stays practically the same and the bank value does not change. The lag in disclosure is not beneficial.

A disclosure lag is not beneficial when the asset has a long duration (Low r and high μ) because the equilibrium asset risk and run threshold already guarantee a low run probability. Even more, if the disclosure lag is big, the bank value decreases as the manager can take more risk helped by the lower run cutoff. The disclosure lag decreases the equilibrium run-threshold but the manager's optimal response is to take advantage of this situation by increasing risk. Even when the lag initially decreases the probability of a run, the endogenous choice of the manager offsets this

Figure 17:

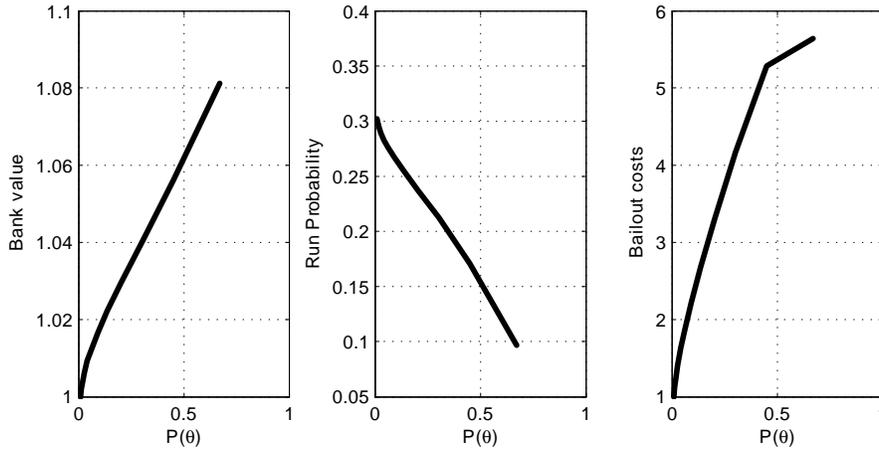


Figure 18: Bank Value, Run Probability and Bailout Costs vs Probability of a Bailout.

benefit and increases the project risk, which in turn increases the default probability.

When the assets have shorter durations, (they have relative higher coupon rate, r , and lower drift, μ) the equilibrium moves to the top right in Figure 16. We are back to the situation presented in Figure 13. Increasing the lag in disclosure produces an equilibrium outcome that has lower risk and higher run cutoff. The reduction in risk effect dominates and there is an increase in the bank value as the lag increases.

A disclosure lag balances the benefits of reduced risk-shifting with the cost of runs when the bank holds short duration assets. Short duration assets have low growth opportunities that motivate the bank manager to compensate by taking on a risky project. This produces an asset with a low growth-to-risk ratio. As the disclosure lag increases, creditors become nervous and tend to run more frequently because of the bad characteristics of the project. The manager optimally reduces risk to compensate the increased probability a bank run. The risk reduction effect dominates and the bank value increases with a disclosure lag.

7 Disclosure lag as an alternative to bailouts

An alternative way to change the run probability is to change the reliability of the bailouts. In the model, the parameter θ measures the intensity at which bankruptcy occurs once there is a run. When $\theta = 0$ the policy is to always bailout the bank whereas when $\theta \rightarrow \infty$ the policy is no-bailout. With the transformation $P(\theta) = e^{-\theta}$ we can give a more intuitive interpretation in terms of probability rather than intensities. A bailing out policy corresponds to $P(0) = 1$ and a no-bailout policy corresponds to $P(\infty) = 0$.

Figure 18 plots the bank value, run probability and bailout costs as the reliability, $P(\theta)$, changes. Moving towards a bailout policy decreases the probability of a run, which in turn increases the bank value. At the same time, the bailout costs increase because the government has made a commitment to save the entity. Bailouts serve as a tool to reduce the occurrence of runs but it is costly. Going from a no-bailout policy to a bailout with a 50% chance increases the costs of bailouts more than five times.

Recall that Figure 12 plots the bank value, run probability and bailout costs as the disclosure lag changes from 0 to 6 years. As the lag increases from 0 to 2 years, the probability of a run decreases and the bank value increases. The bank value attains a maximum and then starts decreasing for values higher than 3 years. The bailout costs decrease and attain a minimum at 2 years. Contrary to the policy of making bailouts more reliable, a disclosure lag decreases bailout costs while decreasing the occurrence of bank runs. That is, a delay in information serves as a policy that saves public resources compared to a more reliable bailout while lowering the probability of a run.

8 Conclusion

This paper studies the effects of disclosure lags on runs and incentive provision for a bank financed with short-term debt. In the model, runs occur because there is an intertemporal coordination problem among creditors as in He and Xiong (2012). Also, there is a risk-shifting problem that allows bank management to have discretion over the asset riskiness. I model the information

disclosure lag explicitly, where creditors receive delayed information about the asset's value. My contribution is threefold. First, I show that when the bank holds assets with low growth rates, a disclosure lag is beneficial because it balances the benefits of reduced risk-shifting with the cost of runs. A disclosure lag increases bank value because the bank manager reduces risk as creditors get nervous with opacity. The risk reduction effect dominates and the bank value increases as the bank run probability decreases.

Second, when the bank's assets have high growth rates, a disclosure lag increases the probability of bank runs and hence decreases the value of the bank. This occurs because in the presence of a disclosure lag, the manager increases asset risk taking advantage of creditors. The increase in asset risk dominates and decreases bank value. Third, assets with high growth rates decrease the manager's appetite for risk and this translates into a reduced run probability and a higher bank value. This occurs because high growth rate assets decrease risk taking by the manager and creditors run less frequently when assets are safe.

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9 Appendix

9.1 Creditor's HJB Equation

In this section we review the creditor's problem and some of its properties. Recall that the creditor's problem can be written as the solution of the following value function equation.

$$\begin{aligned}
 V(x_t) &= E_t \left[\int_t^\tau e^{-\rho(s-t)} r ds + e^{-\rho(\tau-t)} \{ \min(1, y_\tau) \mathbf{1}_{\{\tau=\tau_\phi\}} \right. \\
 &\quad \left. + \min(1, \mathcal{L}(y_\tau)) \mathbf{1}_{\{\tau=\tau_\theta\}} \right. \\
 &\quad \left. + \max_{\text{run or rollover}} (1, V(x_\tau; y_*)) \mathbf{1}_{\{\tau=\tau_\delta\}} \right]
 \end{aligned}$$

Now, we rewrite the problem in a different way. Fix a threshold y_* . At each point in time $u \geq t$, the creditor receives interest payments r and when the project ends he receives $\min(1, x_{u+I})$ or $\min(1, \mathcal{L}(x_{u+I}))$ when the bank is forced into bankruptcy. Therefore, the creditor's expected payoff at time $u \geq t$ is given by

$$\begin{aligned}
 &r + \frac{\phi}{\phi + \theta \delta \mathbf{1}_{\{x_u < y_*\}}} E_u[\min(1, x_{u+I})] \\
 &+ \frac{\theta \delta \mathbf{1}_{\{x_u < y_*\}}}{\phi + \theta \delta \mathbf{1}_{\{x_u < y_*\}}} E_u[\min(1, \mathcal{L}(x_{u+I}))]
 \end{aligned}$$

where $E_u[\min(1, x_{u+I})]$ and $E_u[\min(1, \mathcal{L}(x_{u+I}))]$ have Black-Scholes like formulas. Let $p_1(x, u+I) = E[\min(1, x_{u+I}) | x_u = x]$ and $p_2(x, u+I) = E[\min(1, \mathcal{L}(x_{u+I})) | x_u = x]$ and integrate the discounted payoff across possible values of x to get

$$p(t, x_t) = E_{t, x_t} \left[e^{-\rho(u-t)} \left(r + \frac{\phi}{\phi + \theta \delta \mathbf{1}_{\{x_u < y_*\}}} p_1(x_u, u+I) + \frac{\theta \delta \mathbf{1}_{\{x_u < y_*\}}}{\phi + \theta \delta \mathbf{1}_{\{x_u < y_*\}}} p_2(x_u, u+I) \right) \right].$$

We show that $e^{-\rho t} p(x_t, t)$ is a martingale under P. Let $s \leq t \leq u$

$$\begin{aligned}
& E [p(t, x_t) | F(s)] \\
= & E \left[e^{-\rho t} E \left[e^{-\rho(u-t)} \left(r + \frac{\phi}{\phi + \theta \delta \mathbf{1}_{\{x_u < y_*\}}} p_1(x_u, u + I) + \frac{\theta \delta \mathbf{1}_{\{x_u < y_*\}}}{\phi + \theta \delta \mathbf{1}_{\{x_u < y_*\}}} p_2(x_u, u + I) \right) | F(t) \right] | F(s) \right] \\
= & E \left[E \left[e^{-\rho u} \left(r + \frac{\phi}{\phi + \theta \delta \mathbf{1}_{\{x_u < y_*\}}} p_1(x_u, u + I) + \frac{\theta \delta \mathbf{1}_{\{x_u < y_*\}}}{\phi + \theta \delta \mathbf{1}_{\{x_u < y_*\}}} p_2(x_u, u + I) \right) | F(t) \right] | F(s) \right] \\
= & p(s, x_s).
\end{aligned}$$

by the tower property. Hence, $p(x_t, t)$ is a martingale and it satisfies the Black-Scholes formula.

The creditor's value can be written as the Laplace transform of $p(t, x_t)$, or

$$V(x_t; y_*) = \int_0^\infty (\phi + \theta \delta \mathbf{1}_{\{x_u < y_*\}}) e^{-(\phi + \theta \delta \mathbf{1}_{\{x_u < y_*\}})u} p(t, x_t) du$$

and hence we can apply the Laplace transform to both sides of the Black-Scholes PDE to get equation (7).

9.2 Manager's HJB Equation

In this section we show that the second part in the manager's problem, $Q^2(y_t; y_*, \Gamma)$, satisfies equation (11). Recall that $Q^2(\cdot)$ represents the value for the manager after $t + I$. Mathematically,

$$Q^2(y_t; y_*, \Gamma) = E[e^{-\rho(\tau-t)} \{(y_\tau - 1)^+ \mathbf{1}_{\{\tau=\tau_\phi\}} + (\mathcal{L}(y_\tau) - 1)^+ \mathbf{1}_{\{\tau=\tau_\theta\}}\} | A_2].$$

Fix a time $u \geq t + I$. At time u , the project could expire with payoff $(y_u - 1)^+$, the bank could be forced into bankruptcy with payoff $(\mathcal{L}(y_u) - 1)^+$ or it could continue.

Creditors' receive information with a lag and hence the intensity relevant at time u depends on the value of the fundamental at time $u - I$. The expected payoff at time u can be therefore written as

$$(\phi(y_u - 1)^+ + \theta \delta \mathbf{1}_{\{y_{u-I} < y_*\}} (\mathcal{L}(y_u) - 1)^+)$$

where ϕ represents the intensity of the project expiring at time u and $\theta\delta\mathbf{1}_{\{y_{u-I} < y_*\}}$ represents the intensity of the bank failing at time u .

Consider the discounted expected payoff at time $u - I$

$$payoff(u - I, y_{u-I}) = E_{u-I}[e^{-\rho I}((\phi(y_u - 1)^+ + \theta\delta\mathbf{1}_{\{y_{u-I} < y_*\}}(\mathcal{L}(y_u) - 1)^+))]$$

which can be written as

$$\begin{aligned} payoff(u - I, y_{u-I}) &= \phi E_{u-I}[e^{-\rho I}(y_u - 1)^+] \\ &\quad + \theta\delta\mathbf{1}_{\{y_{u-I} < y_*\}} E_{u-I}[e^{-\rho I}(\mathcal{L}(y_u) - 1)^+] \end{aligned}$$

The expected discounted payoff at time t is therefore

$$c(t, y_t) = E_t[e^{-\rho(u-I-t)} payoff(u - I, y_{u-I})].$$

We show that $e^{-\rho t}c(t, y_t)$ is a martingale under P. Let $s \leq t \leq u - I$ and consider

$$\begin{aligned} &E[e^{-\rho t}c(t, y_t) | F(s)] \\ &= E[E[e^{-\rho(u-I)} payoff(u - I, y_{u-I}) | F(t)] | F(s)] \\ &= E[e^{-\rho(u-I)} payoff(u - I, y_{u-I}) | F(s)] \\ &= e^{-\rho s}c(s, y_s) \end{aligned}$$

which indeed means that $c(\cdot)$ is a martingale. Therefore $c(s, x)$ satisfies the Black-Scholes formula

$$-\rho c(s, x) + c_t(s, x) + \mu x c_x(s, x) + \frac{1}{2} \sigma^2 x^2 c_{xx}(s, x) = 0$$

with boundary conditions

$$\begin{aligned}
c(u - I, x) &= \text{payoff}(u - I, x) \quad \forall x \in [0, \infty) \\
c(s, 0) &= 0 \quad \forall s \in [t, u - I] \\
\lim_{x \rightarrow \infty} c(s, x) &= \phi(xe^{(\mu-\rho)(u-s)} - e^{-\rho(u-s)}) \quad \forall s \in [0, u - I].
\end{aligned}$$

It is convenient to change the time variable to the time to expiration $\tau = u - I - t$ so that $g(\tau, x) = c(u - I - t, x)$ satisfies the PDE

$$-\rho g(\tau, x) - g_\tau(\tau, x) + \mu x g_x(\tau, x) + \frac{1}{2} \sigma^2 x^2 g_{xx}(\tau, x) = 0 \quad (20)$$

with boundary conditions

$$\begin{aligned}
g(0, x) &= \text{payoff}(u - I, x) \quad \forall x \in [0, \infty) \\
g(s, 0) &= 0 \quad \forall s \in [0, u - I] \\
\lim_{x \rightarrow \infty} g(s, x) &= \phi(xe^{(\mu-\rho)(s+I)} - e^{-\rho(s+I)}) \quad \forall s \in [0, u - I].
\end{aligned}$$

Since the maturity or bank failure are exponentially distributed, the manager's value is the Laplace transform of $g(\tau, x)$, or

$$V(y, y_*) = \int_0^\infty e^{-(\phi + \theta \delta \mathbf{1}_{\{y_{u-I} < y_*\}})(u-I)} g(\tau, x) d(u - I)$$

and hence we can apply the Laplace transform to both sides of the Black-Scholes PDE (equation (20)) to get

$$-\rho G(x) - ((\phi + \theta \delta \mathbf{1}_{\{y_{u-I} < y_*\}}) G(x) - g(0, x)) + \mu x G_x(x) + \frac{1}{2} \sigma^2 x^2 G_{xx}(x) = 0$$

where $G(\cdot)$ is the Laplace transform of $g(\tau, x)$.

Finally, replace the boundary condition for $g(0, x)$ to get equation (11)

$$\begin{aligned} \rho G(x) &= \mu x G_x(x) + \frac{1}{2} \sigma^2 x^2 G_{xx}(x) + \phi (E_{u-I}[e^{-\rho I} (y_u - 1)^+] - G(x)) \\ &\quad + \theta \delta \mathbf{1}_{\{y_{u-I} < y_*\}} (E_{u-I}[e^{-\rho I} (\mathcal{L}(y_u) - 1)^+] - G(x)) \end{aligned}$$

9.3 Asset Duration

The average time when cashflows are received (duration) can be computed as

$$E \left[\int_0^\tau s e^{-\rho s} r ds + \tau_\phi e^{-\rho \tau_\phi} y_{\tau_\phi} \right] = \frac{r}{(\rho + \phi)^2} + \frac{\phi}{(\rho + \phi - \mu)^2} y_0$$

which is also the derivative of the asset value with respect to ρ .

If we divide by the value of the assets we get the average maturity

$$\begin{aligned} \text{Duration} &= \frac{\frac{r}{(\rho + \phi)^2} + \frac{\phi}{(\rho + \phi - \mu)^2} y_0}{\frac{r}{(\rho + \phi)} + \frac{\phi}{(\rho + \phi - \mu)} y_0} \\ &= \frac{\frac{r}{(\rho + \phi)^2}}{\frac{r}{(\rho + \phi)} + \frac{\phi}{(\rho + \phi - \mu)} y_0} + \frac{\frac{\phi}{(\rho + \phi - \mu)^2} y_0}{\frac{r}{(\rho + \phi)} + \frac{\phi}{(\rho + \phi - \mu)} y_0} \\ &= \frac{\frac{r}{(\rho + \phi)^2}}{\frac{r(\rho + \phi - \mu) + (\rho + \phi)\phi y_0}{(\rho + \phi)(\rho + \phi - \mu)}} + \frac{\frac{\phi}{(\rho + \phi - \mu)^2} y_0}{\frac{r(\rho + \phi - \mu) + (\rho + \phi)\phi y_0}{(\rho + \phi)(\rho + \phi - \mu)}} \\ &= \frac{r(\rho + \phi)(\rho + \phi - \mu)}{(r(\rho + \phi - \mu) + (\rho + \phi)\phi y_0)(\rho + \phi)^2} + \frac{\phi y_0(\rho + \phi)(\rho + \phi - \mu)}{(r(\rho + \phi - \mu) + (\rho + \phi)\phi y_0)(\rho + \phi - \mu)^2} \\ &= \frac{r(\rho + \phi - \mu)}{(r(\rho + \phi - \mu) + (\rho + \phi)\phi y_0)(\rho + \phi)} + \frac{\phi y_0(\rho + \phi)}{(r(\rho + \phi - \mu) + (\rho + \phi)\phi y_0)(\rho + \phi - \mu)} \\ &= \frac{r(\rho + \phi - \mu)^2 + \phi y_0(\rho + \phi)^2}{(r(\rho + \phi - \mu) + (\rho + \phi)\phi y_0)(\rho + \phi)(\rho + \phi - \mu)} \\ &= \frac{r(\rho + \phi - \mu)^2 + \phi y_0(\rho + \phi)^2}{r(\rho + \phi - \mu)^2(\rho + \phi) + \phi y_0(\rho + \phi)^2(\rho + \phi - \mu)} \end{aligned}$$

9.4 Government's expected bailout costs

Following Cheng and Milbradt (2011) we can compute the Government's expected bailout costs by solving the following ODE.

$$\begin{aligned}\rho G(y_t; y_*) &= \mu y_t G_y + \frac{\sigma^2}{2} y_t^2 G_{yy} - \phi G(y_t; y_*) \\ &\quad - \theta \delta \mathbf{1}_{\{y_t < y_*\}} G(y_t; y_*) \\ &\quad + \delta \mathbf{1}_{\{y_t < y_*\}} [1 - V(y_t; y_*)]\end{aligned}$$

In standard form

$$\begin{aligned}&\frac{\sigma^2}{2} y_t^2 G_{yy} + \mu y_t G_y - G(\rho + \phi + \mathbf{1}_{\{y_t < y_*\}} \theta \delta) \\ &= -\delta \mathbf{1}_{\{y_t < y_*\}} [1 - V(y_t; y_*)]\end{aligned}$$

with boundary conditions

$$\begin{aligned}\lim_{y \rightarrow \infty} G(y) &= 0 \\ G(0) &= \frac{\delta(1 - V(0))}{\rho + \phi + \theta \delta}\end{aligned}$$

where $V(0)$ is the value for the creditors when the fundamental is 0.